On the optimal design of a Financial Stability Fund*

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Abstract

A Financial Stability Fund set by a union of sovereign countries (e.g. the *European Stability Mechanism*) can improve countries' ability to share risks, borrow and lend, with respect to the standard instrument: sovereign debt financing. Efficiency gains arise from the ability of the fund to offer long-term contingent financial contracts, subject to limited enforcement and moral hazard constraints. In contrast, debt contracts are subject to untimely debt roll-overs and default risk. We develop a model of the *Financial Stability Fund* (*FSF*) as a long-term partnership with limited commitment (limited ex-post transfers). We quantitatively compare the constrained-efficient *FSF* economy with the incomplete markets economy with default. In particular, we characterize how (implicit) interest rates and asset holdings differ, as well as how both economies react differently to the same productivity and government expenditure shocks. In our calibrated economies (to world TFP series) there are important efficiency gains in establishing a well-designed *Financial Stability Fund*; particularly, when economies experiment negative shocks. Our theory provides a basis for the design of a *FSF* and a theoretical framework to assess similar mechanisms (e.g. the combination of the *ESM* and the *Outright Monetary Transactions* ECB mechanism).

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1 Introduction

There is ample experience on how funds, such as the IMF, can help to resolve sovereign debt problems. Almost surprisingly, little theoretical research has been done on how a fund set by a group of countries – such as the *European Stability Mechanism* – should be designed and operated. For example, the latter has been operational since October 8, 2012 and its treaty (Ch. 4 Art. 12.1) establishes as its first principle that:

> If indispensable to safeguard the financial stability of the euro area as a whole and of its Member States, the ESM may provide stability support to an ESM Member subject to strict conditionality, appropriate to the financial assistance instrument chosen.

While this first principle assess the need for contingent fund contracts it also limits its funding to 'indispensable' events. Conditionality is a property of the optimal fund contract that we characterize. In principle, a *Financial Stability Fund* (*FSF*) should neither be limited to act as a 'rescue fund,' nor should be expanded to act as a 'transfer fund'. We consider a *FSF* that can implement a long-term contingent fund contract which can act as substitute to other forms of financing, such as sovereign debt financing. But we also take into account that sovereign countries may renege the fund contract and that the redistribution capacity of the *FSF* may be constrained. In other words, we model the *FSF* as a long-term partnership with double-sided limited commitment. We then compare an economy with access to a *FSF*, which in turn has access to international financial markets, with an incomplete markets economy with debt-financing and direct access to the international financial markets. Except for this difference in their financial regimes, both economies have the same characteristics (including exogenous shocks).

Our model of the *FSF* as a partnership builds on the literature on dynamic optimal contracts with enforcement constraints (e.g. Marcet and Marimon (2011)), as well as on the related literature on price decentralization of optimal contracts (e.g. Alvarez and Jermann (2000), Krueger *et al.* (2008)). Our benchmark incomplete markets economy with default builds on the literature on sovereign debt (e.g. Arellano (2008), Aguiar and Gopinath (2006)).

Our model adresess three basic issues: i) debt contracts, while being optimal in anonymous relationship are inefficient in the context of long-term relationships among partner countries; ii) even in the context of a fiscal and monetary union, limited enforceability is a characteristic of sovereign debt and places limits on the amount of persistent redistribution that member countries are willing to tolerate; iii) moral hazard must also be accounted for in guaranteeing solvency. In this version of the paper, we analyze the first two aspects, leaving for future work the *moral hazard problem* (building on Atkeson (1991)).

In order to properly compare the *FSF* economy with the incomplete markets economy we 'decentralize' the fund contract generating the appropriate prices. For example, both, in the *incomplete markets* economy with default and in the two-sided limited *FSF* economy interest rates may differ from the risk-free rate. In the former, the *negative spreads* reflect the risk of default, while in the latter the *positive spreads* reflect the risk that the lender's participation constraint becomes binding. Lower interest rates deter the lender from lending, implementing the *FSF* lender's enforcement constraint . Default results in autarky, with a small probability to return to the fund. In the *incomplete markets* economy voluntary default occurs when the cost of repaying the debt is higher than the cost of getting into autarky, with a low probability to return to the bond market. In contrast, in an *FSF* economy countries do not renege the fund contract in equilibrium: a manifestation of the enhanced efficiency of the *FSF*.

Although for simplicity we analyze the polar case here, the *FSF* could also be a complementary mechanism to debt financing[1]. In this context a main advantage for countries to participate in the fund is that their – possibly, non sustainable – short-term and non state-contingent debt is transformed into a state-contingent long-term debt. In this sense, the fund provides a technology of maturity transformation. In our model, the fund, acting as a risk-neutral lender, can freely borrow and lend in the international markets at a risk-free interest rate, while providing long-term conditional financing to the borrower.

Our economies are subject to technology and government expenditure shock processes. The former is assumed to be a Markov Switching process that has been estimated using world TFP by Bai and Zhang (2010). At this point we do not have a proper data set of world government expenditures and we assume this process follows a three-state Markov process, with relatively low probability for the higher liability state. In this sense, our *FSF* can be viewed as a worldwide fund, in a world of no global uncertainty and relatively small sovereign countries.

Computing our economies allows for close inspection of the policy functions, showing how different regimes result in different consumption, labor and asset holdings decisions. In particular, how the same sequence of productivity and government shocks differently affects agents' decisions, resulting in different equilibrium paths across regimes. These differences in policies also translate, in our computed economies, in substantial welfare gains from implementing a properly designed *FSF*.

We also show how these economies differently react to a permanent, as well as to a transitory shock. Our economies with relatively more impatient risk-averse agents (countries), with separable disutility for labor, contracting with risk-neutral agents have clear, and known, efficiency benchmarks: consumption of the borrowing country should decay smoothly and effort should be positively correlated with productivity shocks (higher effort when it is more productive). Economies with limited enforcement constraints can be relatively close to such benchmarks, except that enforcement constraints deter from consumption decay or may force agents to work harder in low productivity states. Nevertheless, distortions are typically larger in the economies with incomplete markets; particularly so when default risk is high. Our computed exercises provide clear

[1]Central bank sovereign bond market interventions, such as the ECB interventions in the euro crisis, also complement normal debt financing from households.

images of these differences. For example, of how an *FSF* economy provides better financing opportunities to the borrower at the same cost for the lender, or better copes with shocks that result in default in the incomplete markets economy. Crisis amplify the gains from establishing a *FSF*, as opposed to simply relying on sovereign debt.

2 Economies with different borrowing possibilities

We consider a standard infinite-horizon representative agent economy, where the agent has preferences for current leisure, $l = 1 - n$, and consumption, c, represented by $u(c, 1 - n)$ and discounts the future at the rate β. The agent has access to a decreasing returns labor technology $y = \theta f(n)$, where $f' > 0$, $f'' < 0$ and θ is a productivity shock, assumed to be Markovian; $\theta \in (\theta_1, ..., \theta_N)$, $\theta_i < \theta_{i+1}$. The economy is a small open economy in a world with no uncertainty with interest rate r satisfying $1/(1 + r) \geq \beta$; an inequality that, in general, we will assume to be strict. The country is also subject to a government expenditure shock G, which follows a partially endogenous joint stochastic process with productivity θ. In particular, $G \in (G_1, ..., G_M)$, with $G_j > G_{j+1}$. The exogenous state is denoted by $s = (\theta, G)$ and s^t the history of the realizations of both shocks up to period t. The joint transition probability is denoted by $\pi(s'|s)$, where π is a $K \times K$ transition matrix and $K = M \times N^2$. Economies only differ by their financial structures.

2.1 Incomplete markets

In the *incomplete markets* economy agents have only one financial instrument, one-period non-contigent debt, to borrow and lend with the possibility of default, denoted (IMD), as in the economy analyzed by Arellano (2008)[3]. The agent's problem has the following recursive form:

$$V_b^i(b, s) = \max_{c,n,b'} \left\{ u(c) + U(1 - n) + \beta \mathrm{E} \left[V_b^{ia}(b', s') \mid s \right] \right\}$$

$$\text{s.t.} \quad c + G + q(s, b')b' \leq \theta f(n) + b,$$

where c is consumption, n is labour, b are the ***asset holdings*** at the beginning of the period (i.e. $-b'$ is the amount of new one-period debt being borrowed), and $q(s, b')$ is the ***price of one-period bond***, which is conditional on the amount being borrowed and the current state, but not on the state of next period. The value function satisfies:

$$V_b^{ia}(b, s) = \max\{V_b^i(b, s), V^{ai}(s)\},$$

[2] We will introduce the possibility that costly effort, e, affects the distribution of G and, therefore, the transition π in later versions.

[3] Introducing the dependence of the distribution of G on effort, e, will result in a third 'moral hazard' regime.

where V^{ai} denotes the value of reverting to autarky. In this case, there is a probability $1-\lambda$ that the country stays in autarky and a probability of $\lambda \in [0,\, 1)$ that the country has access to the one-period bond market even if by defaulting debt liabilities are reduces to zero, as we assume. Nevertheless, we also assume that λ is low enough as to not to deter borrowing (i.e. to avoid Bulow-Rogoff's problem). More precisely, the value in autarky is given by

$$V^{ai}(s) = \max_{n} \{u((\theta f\,(n) - G) + U(1-n)$$
$$+\beta \mathrm{E}\left[(1-\lambda)\, V^{ai}(s') + \lambda V_b^i\,(0, s') \mid s\right]\}$$

The choice of default is given by

$$D(s,b) = 1 \text{ if } V^{ai}(s) > V_b^i(b,s) \text{ and } 0 \text{ otherwise.}$$

Let the expected default rate be denoted by $d(s,b') = \mathrm{E}\left[D(s',b') \mid s\right]$, then the price of new debt is $q(s,b') = \frac{1-d(s,b')}{1+r}$ and, therefore, the debt interest rate is $r^i(s,b') = 1/q(s,b') - 1$ and the resulting ***spread*** is $r^i\,(s,b') - r \geq 0$.

A special case are the 'Bewley economies' where the enforcement technology deters agents from ever defaulting, denoted (IM). By definition, in Bewley economies $d(s,b') = 0$, therefore: $q(s,b') = q = 1/(1+r)$.

A *recursive competitive equilibrium* for these *incomplete markets* economies is defined in the standard way as a set of policy functions: $c_b^i(s,b)$, $n^i(s,b), b^{i\prime}(s,b)$, $D(s,b)$ and value functions, V_b^i, V_b^{ai}, that solve the borrower's problem for the corresponding bond prices, $q(s,b')$. As usual, this partial equilibrium definition can be extended to a general equilibrium definition by adding risk-neutral lenders, with a rate of time preference r, who could freely enter the market in period zero.

Finally, in order to keep track of debt flows, it is useful to define some basic debt accounting measures. In particular, the ***pimary surplus*** – or *primary deficit* when it is negative – is given by

$$q(s,b')b' - b = \theta f\,(n) - (c+G), \tag{1}$$

and the ***surplus*** (primary surplus + interest repayment), at the end of the period[4], is given by

$$b' - b = q(s,b')b'(1 + r^i(s,b')) - b. \tag{2}$$

2.2 Economies with a *Financial Stability Fund*

An economy with a *Financial Stability Fund* (*FSF*) is modeled as a long-term contract between the representative agent of a small open economy and a risk-neutral lender, who can freely borrow and lend in the international market. The *fund contract* establishes that the agent (the borrowing country) consumes c and the resulting transfer to the *FSF* (the lender) is $\tau = \theta f\,(n) - (c+G)$;

[4]For accounting convenience, we consider 'end-of-period' interest payments.

i.e. when $\tau < 0$ the country is effectively borrowing. In general we consider *FSF* economies where both the borrrower and the lender can renege the fund contract. If the country quits the fund in period t it keeps all the output $\theta_t f(n_t)$ and with probability $1-\lambda$ remains in autarky and with probability $\lambda \in [0,1)$ is admitted back into the fund. The value of such outside option is $V^{af}(s)$. If the lender quits the fund in period t its outside option value is $Z \leq 0$; that is, in general we consider economies with *two-sided limited enforcement*, denoted (2S). A special case are the *one-sided limited enforcement* economies, denoted (1S), where the lender participates with full commitment to the fund as long as the initial value of fund contract is at least $Z \leq 0$.

The outside option of the country satisfies the the following Bellman equation:

$$\begin{aligned} V^{af}(s) &= \max_n \{u((\theta f(n) - G) + U(1-n) \\ &+\beta \mathrm{E}\left[(1-\lambda) V^{af}(s') + \lambda V_b^f(0, s') \mid s\right]\} \end{aligned} \tag{3}$$

where $V_b^f(s)$ is the borrower's value of starting a new fund contract in state s, which we will now define[5].

The optimal fund contract is a solution to the following problem:

$$\begin{aligned} \max_{\{c(s^t), n(s^t)\}} \quad & \mathrm{E}\left[\mu_{b,0} \sum_{t=0}^{\infty} \beta^t \left[u(c(s^t)) + U(1-n(s^t))\right] \right. \\ & \left. + \mu_{l,0} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \tau(s^t) \mid s_0 \right] \\ \text{s.t.} \quad & \mathrm{E}\left[\sum_{r=t} \beta^{r-t} \left[u(c(s^r)) + U(1-n(s^r))\right] \mid s^t\right] \geq V^{af}(s_t) \qquad (4) \\ & \mathrm{E}\left[\sum_{r=t} \left(\frac{1}{1+r}\right)^{r-t} \tau(s^t) \mid s^t\right] \geq Z, \qquad (5) \\ \text{and} \quad & \tau(s^t) = \theta(s^t) f\left(n(s^t)\right) - c(s^t) - G(s^t), \quad t \geq 0. \end{aligned}$$

where (4) and (5) are the intertemporal participation constraints for the borrower and the lender, respectively, and (μ_{b0}, μ_{l0}) are initial Pareto weights that, without loss of generality, we normalize $\mu_{b0} = 1$ and μ_{l0} as to satisfy (5) in period zero with equality. The *FSF* contract in an economy with *one-sided limited enforcement* is a solution to the same problem with (5) only being a constraint in period zero, while the *First Best* (unconstrained) solution is achieved when both (4) and (5) are at most binding in period zero, denoted (FB).

As it is known (see Marcet and Marimon, 2012) we can rewritte the fund contract problem as:

[5]We will also consider the possibility that a country joins the fund with a debt liability, but to simplify the exposition here we only consider fund contracts initiated with zero debt

$$\min_{\{\gamma_{b,t},\gamma_{l,t}\}} \max_{\{c_t,n_t\}} \mathrm{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\mu_{b,t+1}\left[u(c_t)+U(1-n_t)\right]-\gamma_{b,t}V^A(s_t)\right)\right.$$
$$\left. + \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t \left(\mu_{l,t+1}\tau_t-\gamma_{l,t}Z\right) \mid s_0\right]$$
$$\mu_{i,t+1}(s^{t+1}) = \mu_{i,t}(s^t)+\gamma_{i,t}(s^t)\,,\ \mu_{i,0}(s_0) \text{ is given, for } i=b,l,$$

where $\gamma_b(s^t)$ and $\gamma_l(s^t)$ are the Lagrange multipliers of the enforcement constraints (4) and (5), respectively, in state s^t. That is, with one-sided limited commitment $\gamma_l(s^t)=0$, $\forall t\geq 0$. Let $\eta \equiv \beta(1+r)\leq 0$, then the first-order conditions of this problem are:

$$u'(c(s^t)) = \frac{\mu_l(s^{t+1})}{\mu_b(s^{t+1})\,\eta^t}, \tag{6}$$
$$\frac{U'(1-n(s^t))}{u'(c(s^t))} = \theta(s^t)f'\left(n\left(s^t\right)\right). \tag{7}$$

We can normalize the multipliers by defining

$$x(s^{t+1}) = \frac{\mu_l(s^{t+1})}{\mu_b(s^{t+1})\,\eta^t} \text{ and } v_i(s^t)=\frac{\gamma_i(s^t)}{\mu_i(s^t)}, i=l,b;$$

$x(s^{t+1})$ is the temporary relative Pareto weight of the lender, and $v_b(s^t)$ and $v_l(s^t)$ are the normalized Lagrange multipliers of the agents. Notice that with this normalization (6) simplifies to: $u'(c(s^{t+1}))=x(s^{t+1})$ and, furthermore,

$$x(s^{t+1}) = \frac{1}{\eta}\frac{1+v_l(s^t)}{1+v_b(s^t)}x\left(s^t\right).$$

Since we have assumed that exogenous shock processes are Markovian, it is easy to see that the first order conditions have a recursive structure, corresponding to the recursive formulation of the fund contract, which defines optimal policies – $c^*(x,s), n^*(x,s)$ and $v_b(x,s), v_l(x,s)$ – satisfying:

$$u'(c^*(x,s)) = x' = \frac{1}{\eta}\frac{1+v_l(x,s)}{1+v_b(x,s)}x \tag{8}$$

and

$$\frac{U'(1-n^*(x,s))}{u'(c^*(x,s))} = \theta f'(n^*(x,s)). \tag{9}$$

The value function of the fund contract saddle-point problem takes the form: $W(x,s)=xV_l^f(x,s)+V_b^f(x,s)$, where

$$V_b^f(x,s) = u(c^*(x,s))+U(1-n^*(x,s))+\beta\sum_{s'\in S}\pi(s'|s)V_b^f(x',s') \tag{10}$$

and

$$V_l^f(x,s) = \tau^*(x,s) + \frac{1}{1+r}\sum_{s'\in S}\pi(s'|s)V_l^f(x',s'), \tag{11}$$

with $\tau^*(x,s) = \theta f\left(n^*(x,s)\right) - \left(c^*(x,s)+G\right)$.

The optimal policy functions are determined by the first order conditions (8) and (9), together with the following slackness conditions

$$V_b^f(x,s) \geq V^{af}(s) \text{ and } v_b(x,s)\left[V_b^f(x,s) - V^{af}(s)\right] = 0 \tag{12}$$

and

$$V_l^f(x,s) \geq Z \text{ and } v_l(x,s)\left[V_l^f(x,s) - Z\right] = 0, \tag{13}$$

Finally, recall that, (x_0, s_0) is defined by $x_0 = \mu_{l0} = \mu_l(s_0)$ such that $V_l^f(x_0,s_0) = Z$.

3 Decentralization of the fund contract

We now show how to decentralize the optimal contract as a competitive equilibrium with endogenous borrowing constraints, which will allow us to compare the different fund contracts with the debt contract of the economy with incomplete markets. We build on the work of Alvarez and Jermann (2000) and Krueger, Lustig and Perri (2008).

3.1 The competitive equilibrium

In the market equilibrium, the borrower has a home technology that produces $\theta(s^t)f\left(n(s^t)\right)$ with his own labor. The borrower has access to a complete set of one period Arrow securities and solves the following problem:

$$\max_{\left\{c_b(s^t),n(s^t),a_b\left(s^{t+1}\right)\right\}} \sum_{t=0}\sum_{s^t}\beta^t\pi\left(s^t\right)\left[u(c_b(s^t)) + U(1-n(s^t))\right]$$

$$\text{s.t. } c_b(s^t) + \sum_{s^{t+1}|s^t} q\left(s^{t+1}|s^t\right)a_b\left(s^{t+1}\right) = \theta(s^t)f\left(n(s^t)\right) - G(s^t) + a_b(s^t)$$

$$a_b\left(s^{t+1}\right) \geq A_b\left(s^{t+1}\right)$$

where $q\left(s^{t+1}|s^t\right)$ is the price of the one period state contingent claim and $a_b\left(s^{t+1}\right)$ represents the amount of state contingent claims chosen by the borrower and $A_b\left(s^{t+1}\right)$ is an endogenous borrowing limit defined below. The Euler and transversality conditions imply that:

$$q\left(s^{t+1}|s^t\right) \geq \beta^t\pi\left(s_{t+1}|s_t\right)\frac{u'\left(c_b\left(s^{t+1}\right)\right)}{u'\left(c_b\left(s^t\right)\right)}$$

with equality if $a_b\left(s^{t+1}\right) > A_b\left(s^{t+1}\right)$ and

$$\lim_{t\to\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)u'\left(c_b\left(s^t\right)\right)\left[a_b\left(s^t\right) - A_b\left(s^t\right)\right] \leq 0$$

The lender also has access to a complete set of Arrow securities and he solves:

$$\begin{aligned}
&\max_{\{c_l(s^t),a_l(s^{t+1})\}} \sum_{t=0}^{\infty}\sum_{s^t}\left(\frac{1}{1+r}\right)^t \pi\left(s^t\right)c_l(s^t) \\
&\text{s.t. } c_l(s^t)+\sum_{s^{t+1}|s^t} q\left(s^{t+1}|s^t\right)a_l\left(s^{t+1}\right)=a_l(s^t) \\
&a_l\left(s^{t+1}\right)\geq A_l\left(s^{t+1}\right)
\end{aligned}$$

As above, the Euler and transversality conditions imply:

$$q\left(s^{t+1}|s^t\right)\geq\left(\frac{1}{1+r}\right)^t \pi\left(s_{t+1}|s_t\right)$$

with equality if $a_l\left(s^{t+1}\right)>A_l\left(s^{t+1}\right)$ and

$$\lim_{t\to\infty}\sum_{s^t}\left(\frac{1}{1+r}\right)^t \pi\left(s^t\right)\left[a_l\left(s^t\right)-A_l\left(s^t\right)\right]\leq 0$$

Market clearing implies that:

$$\begin{aligned}
c_b\left(s^t\right)+c_l\left(s^t\right) &= \theta(s^t)f\left(n(s^t)\right)-G\left(s^t\right)\text{ for all } s^t \\
a_b\left(s^{t+1}\right)+a_l\left(s^{t+1}\right) &= 0 \text{ for all } s^{t+1}
\end{aligned}$$

The values for the borrower and the lender in the trading arrangement can be writen recursively as

$$\begin{aligned}
W_b(a_b\left(s^t\right),s^t) &= u(c_b(s^t))+U(1-n(s^t))+\beta\sum_{s^{t+1}|s^t}\pi(s_{t+1}|s_t)W_b(a_b\left(s^{t+1}\right),s^{t+1}) \\
W_l(a_l\left(s^t\right),s^t) &= c_l(s^t)+\frac{1}{1+r}\sum_{s^{t+1}|s^t}\pi(s_{t+1}|s_t)W_l(a_l\left(s^{t+1}\right),s^{t+1})
\end{aligned}$$

We assume that the borrowing limits are properly tight in the sense that satisfy:

$$W_b(A_b\left(s^t\right),\theta(s^t)) = V^{af}(s^t) \tag{14}$$

$$W_l(A_l\left(s^t\right),\theta(s^t)) = Z \tag{15}$$

Let

$$Q\left(s^t|s_0\right)=q\left(s^1|s_0\right)q\left(s^2|s^1\right)...q\left(s^t|s^{t-1}\right)$$

We consider allocations that satisfy the *high implied interest rate condition*, namely:

$$\sum_{t=0}^{\infty}\sum_{s^t}Q\left(s^t|s_0\right)\left[c_b\left(s^t\right)+c_l\left(s^t\right)\right]<\infty$$

3.2 Decentralization

Let $\{c_b^*(s^t), c_l^*(s^t), n^*(s^t)\}$ be the allocations in the optimal fund contract. We now show that we can decentralize them as a competitive equilibrium with endogenous borrowing constraints that are not too tight. First, we use the allocations to define the Arrow security price as follows:

$$q^*\left(s^{t+1}|s^t\right) = \beta\pi\left(s_{t+1}|s_t\right)\frac{u'\left(c_b^*\left(s^{t+1}\right)\right)}{u'\left(c_b^*\left(s^t\right)\right)} \text{ if } v_b\left(s^{t+1}\right)=0 \text{ and } v_l\left(s^{t+1}\right)\geq 0;$$

$$q^*\left(s^{t+1}|s^t\right) = \frac{1}{1+r}\pi\left(s_{t+1}|s_t\right) \text{ if } v_l\left(s^{t+1}\right)=0 \text{ and } v_b\left(s^{t+1}\right)>0.$$

Note that the price can be expressed alternatively as follows (see Appendix for details):

$$\begin{aligned} q^*\left(s^{t+1}|s^t\right) &= \max\left\{\beta\pi\left(s_{t+1}|s_t\right)\frac{u'\left(c^*\left(s^{t+1}\right)\right)}{u'\left(c^*\left(s^t\right)\right)}, \left(\frac{1}{1+r}\right)\pi\left(s^{t+1}|s^t\right)\right\} \qquad (16)\\ &= \max\left\{\beta\pi\left(s_{t+1}|s_t\right)\frac{1+v_l(x_{t+1},s_{t+1})}{(1+v_b(x_{t+1},s_{t+1}))\eta}, \left(\frac{1}{1+r}\right)\pi\left(s_{t+1}|s_t\right)\right\}\\ &= \left(\frac{1}{1+r}\right)\pi\left(s_{t+1}|s_t\right)\max\left\{\frac{1+v_l(x_{t+1},s_{t+1})}{1+v_b(x_{t+1},s_{t+1})},1\right\}. \end{aligned}$$

Since we impose borrowing limits that bind exactly when the participation constraints are binding in the optimal fund contract, $q\left(s^{t+1}|s^t\right)=q^*\left(s^{t+1}|s^t\right)$ satisfies the Euler conditions in the competitive equilibrium.

The prices $q\left(s^{t+1}|s^t\right)$ derived from the allocation of consumption and labor of the optimal fund contract defines the ***price of a one-period bond***:

$$q^f\left(s^t\right)=\sum_{s^{t+1}|s^t} q\left(s^{t+1}|s^t\right)$$

This is the implicit price that we can use to compare with the one-period bond price in the incomplete markets economy. Alternatively, we can use the implicit interest rate: $r^f\left(s^t\right)=1/q^f\left(s^t\right)-1$ or the ***spread***: $r^f\left(s^t\right)-r$.

Notice that, if the lender's participation constraint is not binding at s^{t+1}: $(1+v_b(x_{t+1},s_{t+1}))^{-1}\leq 1$. Therefore, either there is no *spread* – for example, in the (1S) economy – or *the spread is negative.* The latter occurs in the (2S) economy when lender's participation constraint is binding, in some s^{t+1}, as to make $\sum_{s^{t+1}|s^t}\pi\left(s_{t+1}|s_t\right)\left\{\frac{1+v_l(x_{t+1},s_{t+1})}{1+v_b(x_{t+1},s_{t+1})}\right\}>1$; that is, when the market price of the lender's transfer, $\tau(s^t)$, is greater than the international market price at which he borrows and lends. In other words, *the spread* $r^f(s^t)-r<0$ reflects the *the wedge* created by the lender's participation constraint; a wedge that aligns the market price with the lender unwillingness to lend. In particular, *the spread can only be negative when there is no borrowing* (i.e. $r^f\left(s^t\right)-r<0\Longrightarrow \tau\left(s^t\right)\geq 0$).

In what follows, we let

$$Q\left(s^t|s_0\right) = q\left(s^1|s_0\right) q\left(s^2|s^1\right) ...q\left(s^t|s^{t-1}\right)$$

Note also that $n^*\left(s^t\right)$ clearly satisfies the optimality condition in the competitive equilibrium with respect to labor.

We can use the intertemporal budget constraints to construct the ***asset holdings*** that make the allocations in the optimal contract satisfy the present value budget, namely:

$$\begin{aligned}
a_b\left(s^t\right) &= \sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q^*\left(s^{t+n}|s^t\right)\left[c^*\left(s^{t+n}\right) - \left(\theta(s^{t+n})f\left(n^*(s^{t+n})\right) - G\left(s^{t+n}\right)\right)\right] \\
&= -\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q^*\left(s^{t+n}|s^t\right)\tau^*\left(s^{t+n}\right) \\
a_l\left(s^t\right) &= \sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q^*\left(s^{t+n}|s^t\right)c_l\left(s^{t+n}\right) = \sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q^*\left(s^{t+n}|s^t\right)\tau^*\left(s^{t+n}\right) \\
a_l\left(s^t\right) &= -a_b\left(s^t\right).
\end{aligned}$$

In this economy, binding participation constraints provide us with the borrowing limits given by (14) and (15). More precisely,

$$\begin{aligned}
A_b\left(s^t\right) &= -\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q\left(s^{t+n}|s^t\right)\left[\theta(s^{t+n})f\left(n_b^*(s^{t+n})\right) - G\left(s^{t+n}\right)\right] \\
&= \sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q^*\left(s^{t+n}|s^t\right)\left(\tau_b^*\left(s^{t+n}\right) - c_b^*\left(s^{t+n}\right)\right) \\
A_l\left(s^t\right) &\geq Z \\
&= \sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t}\left(\frac{1}{1+r}\right)^t \tau_l^*\left(s^{t+n}\right)
\end{aligned}$$

where the first equality refers to histories $\{s^{t+n}\}_{n=0}^{\infty}$ following a state s^t where the enforcement constraint of the borrower is binding (i.e. the borrower is indifferent between remaining in the *FSF* contract and autarky) and, similarly the inequality corresponds to histories following a state where the enforcement constraint of the lender, who values transfers at the risk-free interest rate, is binding.

The corresponding *recursive competitive equilibrium* for these *FSF* decentralized economies is also defined in the standard way as a set of policy functions: $c_b^f(a_b, s)$, $n^f(a_b, s), a_b'(a_b, s), c_l^f(a_l, s), a_l'(a_l, s)$ and value functions, W_b^f, W_l^f, that solve the agents problems for the corresponding Arrow security prices, $q\left(s'|s\right)$

and markets clear. In particular, the value functions satisfy

$$W_b^f(a_b, s) = u(c_b(a_b, s)) + U(1 - n(a_b, s)) + \beta \sum_{s'} \pi(s'|s) W_f^f(a_b', s') \quad (17)$$

$$W_l^f(a_l, s) = c_l(a_l, s) + \frac{1}{1+r} \sum_{s'} \pi(s'|s) W_l^f(a_l', s'), \quad (18)$$

and $c_l(a_l, s) = \theta f(n(a_b, s)) - c_b(a_b, s) - G$ and $a_l'(a_l, s) = -a_b'(a_b, s)$. Equations (17) and (18) clearly show how the value functions (W_b^f, W_l^f) are the mirror image of the value functions (V_b^f, V_l^f), satisfying (10) and (11), with $c_l(a_l, s) = \tau(x, s)$ and the corresponding change of the endogeneous state variable, taking into account that in equilibrium $a_l = -a_b$; i.e. x and (a_b, a_l) have the same dimension. In sum, it shows why we can use the derived asset prices to diagnose the properties of the optimal fund contract.

Finally, some *FSF* accounting is also useful. Paralleling the discussion of the *incomplete markets,* the ***primary surplus*** (primary deficit if negative) is given by

$$\sum_{s'|s} q\left(s'|s\right) a_b'\left(a_b(s), s'\right) - a_b(s) = c_l(a_l, s) = \tau(x, s),$$

and the ***surplus***, or deficit, (primary surplus plus, end-of-period, interest repayments) is given by

$$\begin{aligned} a_b'\left(a_b(s), s'\right) - a_b(s) &= [\sum_{s'|s} q\left(s'|s\right) a_b'\left(a_b(s), s'\right) - a_b(s)] \\ &+ [a_b'\left(a_b(s), s'\right) - \sum_{s'|s} q\left(s'|s\right) a_b'\left(a_b(s), s'\right)], \end{aligned}$$

while the corresponding ***repayment*** is

$$a_b'\left(a_b(s), s'\right) - \sum_{s'|s} q\left(s'|s\right) a_b'\left(a_b(s), s'\right).$$

4 Solution Method and Parameterization

We solve the models numerically using a policy iteration algorithm. We assume the followinsg utility for the borrower: $\log(c) + \frac{\gamma(1-n)^{1-\sigma}}{1-\sigma}$ and the following production function $f(n) = n^{\alpha}$. The short term contract is a standard incomplete market economy with a fixed borrowing constraints, as described in Subsection 2.1. The fund contract policies and value functions are derived by solving equations (8), (9), (10), (11) – with the corresponding slackness conditions – and (3) (see Appendix for details). Using one-period Arrow securities, the bond price (16) simplifies to:

$$
\begin{aligned}
q\left(s'|s\right) &= \max\left\{\beta\pi\left(s'|s\right)\frac{u'\left(c^*\left(x',s'\right)\right)}{u'\left(c^*\left(x,s\right)\right)},\left(\frac{1}{1+r}\right)\pi(s'|s)\right\} \\
&= \pi\left(s'|s\right)\max\left\{\beta\frac{c^*\left(x,s\right)}{c^*\left(x',s'\right)},\left(\frac{1}{1+r}\right)\right\}
\end{aligned}
$$

The price of a **one period bond** is then equal to:

$$
q^f\left(s\right) = \sum_{s'\in S} q\left(s'|s\right)
$$

which in turn implies a risk free rate of $r^f\left(s\right) = \frac{1}{q^f(s)}$. Finally, we can recover the asset holdings numerically by iterating to find the **asset holding** function that satisfies:

$$
\begin{aligned}
a_b\left(z,s\right) &= \sum_{\theta'\in S} q\left(s',s\right)a_b\left(z',s'\right) + c\left(z,s\right) - \theta f\left(n\left(z,s\right)\right) + \left(1-\phi\right)G \\
a_l\left(z,s\right) &= -a_b\left(z,s\right)
\end{aligned}
$$

Moreover, we define **the repayment** as:

$$
a_b\left(z',\theta'\right) - \sum_{\theta'\in S} q\left(\theta',\theta\right)a_b\left(z',\theta'\right)
$$

4.1 Parameterization

The model period is assumed to be one quarter. To make the different contracts comparable, we choose the same parameter values across economies. The technology shock θ is assumed to be a Markov Switching process that has been estimated using world TFP by Bai and Zhang (2010). More precisely, the authors specify *the world productivity process* as a stochastic regime-switching process that has three regimes, each of which is captured by the mean, persistence and standard deviation of innovations $\left\{\left(\mu_j,\rho_j,v_j\right)\right\}_{j=1,2,3}$. The TFP shock a_{it} of country i at period t in regime j follows an autoregressive process:

$$
a_{it} = \mu_j\left(1-\rho_j\right) + \rho_j a_{it-1} + v_j\varepsilon_{it}
$$

where ε_{it} is independently and identically distributed and drawn from a standard normal distribution. At period $t+1$, country i has some probability of switching to another regime, governed by the transition matrix P. To estimate the parameters of the three regimes as well as the transition probabilities in P, the authors use maximum likelihood. The estimated parameters are displayed below:

Table 1: Parameters of the Markov Switching Process for World TFP

All Countries	μ	v	ρ	Low	Middle	High
Low	2.07	0.023	0.995	0.92	0.04	0.04
Middle	3.46	0.070	0.987	0.06	0.90	0.04
High	4.58	0.020	0.981	0.04	0.03	0.93

Following the authors, the process is discretized into a 9 state Markov chain, with three values in each regime. It should be noticed that, as in Bai and Zhang (2010), the resulting Markovian TFP process is relatively volatile (see v in Table 1). There is no similar estimate for the government shock and, therefore, we have performed different simulation exercises in order to define a process which seems reasonable if one considers G as current government liabilities, other than interest payments on sovereign debt[6]. The government shock is also assumed to be a persistent Markov chain with a relatively small probability of a very bad shock. The transition matrix and government shock values are given below:

$$\begin{aligned}\pi_G &= \begin{bmatrix} 0.9 & 0.067 & 0.033 \\ 0.01 & 0.9 & 0.09 \\ 0.005 & 0.095 & 0.9 \end{bmatrix} \\ G &= \begin{bmatrix} 0.05 & 0.025 & 0 \end{bmatrix}\end{aligned}$$

As stated earlier, we assume that the utility of the borrower is additively separable in consumption and leisure. In particular, for our computations we assume

$$\log(c) + \frac{\gamma(1-n)^{1-\sigma}}{1-\sigma}, \text{ with } \sigma = 2, \gamma = 1,$$

and a discount factor equal to $\beta = 0.96$. The interest rate is set to $r = 0.01$, implying a different discount factor for the lender of $\frac{1}{1+r} = 0.9901$, as well as a growth rate for the relative pareto weight of the borrower of $\eta = 0.9696$ in the optimal contract. Regarding the technology, we assume that $f(n) = n^\alpha$ with the labor share of the borrower set to $\alpha = 0.67$. The participation constraint of the lender in the optimal contract is set to $Z = -0.1$, and we choose a looser value of $Z = -1$ for the borrowing limit in the short term contract so that the long term asset holdings with no default are of similar magnitude to those in the optimal contract. Finally, the probability that the borrower comes back to the market upon default in the optimal contract (or that the borrowing country returns to a *FSF* after reneging from a fund contract) is set to $\lambda = 0$.

5 Numerical Results

This section discusses the numerical results. As said, our calibrations uses the world TFP estimates of Bai and Zhang (2010) and postulates a G process. Therefore, the numerical results are an illustration of the world economy as a global riskless economy composed of small countries who borrow and save either by issuing sovereign debt (one period bonds) or through a worldwide *FSF*, and the latter may be subject to ex-ante and ex-post limits on the amount of transfers. Therefore, we mostly discuss the incomplete markets economy with default (**IMD**) and the economy with a *FSF* with two sided lack of commitment (**2S**), although in some case it is illustrative to also report results for the

[6] Preliminary work with a subset of EU countries (those highly indebted and in most need of the ESM) allows to for a better calibration of G and θ.

incomplete markets economy with no default (**IM**), or the economy with a *FSF* with full commitment (**FB**) or one sided lack of commitment (**1S**). We first present the *policy functions* and then, in Subsection 5.2, we present simulations for three different experiments. Figures 1 - 4 illustrate these results. TFP shocks are labeled $e_i, i = 1, \ldots, 9$ where $e_i < e_{i+1}$ and G shocks are labeled $g_j, ij = 1, \ldots, 9$ where $j_i > e_{j+1}$; that is (e_1, g_1) is the worst combination of shocks and, increasingly, (e_9, g_3) is the best combination of shocks.

5.1 Policy Functions

The core of the analysis is given by the study of the different optimal policy functions. Figure 1 displays the policy functions for the main variables for the incomplete markets economy with default (**IMD**), and the economy with a *FSF* and two sided lack of commitment (**2S**), with (**IM**) and (**FB**) as references. The policies are plotted for selected values of shocks ($s = (\theta,\, G) = (e,\, g)$) in: (**IMD**) as function of the level of debt, and in (**FB**) and (**2S**) as a function of the relative Pareto weight of the borrower, which we denote *pareto weight* in what follows.

In Figure 1, (a) and (b.1), the left panel on the first row depicts the policy function for the borrower's consumption, $c^i(s, b)$ and, clockwise, the corresponding labor, $n^i(s, b)$, and debt, $b^{i\prime}(s, b)$ policies, together with the borrower's value, $V_b^i(b, s)$; (a.1) for (**IM**) and (b.1) for (**IMD**). Without default, (**IM**), consumption and debt choices, as well as borrower's value are monotone with respect to shocks at all levels of debt, while the labor choice losses its monotonicity at high levels of debt (high negative values), showing that when a country is heavily indebted works harder when it is subject to negative productivity shocks. The economy with default (**IMD**) distorts the previous policies by the choice of default, which is shown as a discontinuity in $b^{i\prime}(s, b)$ and a corresponding constant autarkic choice, and value, for higher (more negative) values of (non-contracted) debt in the other panels. It should be noticed that the labor choice reversals occur at levels of debt just below the default level and the 'working harder in bad times' persists in autarky. Figure 1 (b.2) provides more information regarding the default choice. The lower-right panel shows the default threshold, $D(s, b)$, for different G shocks as a function of the θ shocks; that is, the lines indicate values of debt at which default occurs (i.e. for higher values below the line the country prefers to be in autarky). The peculiar shape of g_1 reflect the persistency of the worst case scenario (a low probability event). The other three panels illustrate $b^{i\prime}(s, b)$ for the three different values of G. As it can be seen, at positive values of b' bond price corresponds to the riskless rate (i.e. 0.99), while *spreads*, reflecting default risk, appear for higher (more negative) levels of debt, according to the severity of the shocks; in particular, for levels below the default threshold there is no b' (as it can be seen in the upper-right panel for (e_1, g_1)).

Figure 1(c) gives the first best policies, (**FB**), as benchmarks. As expected, there is perfect consumption risk-sharing, although consumption decays (with the weight decaying at the rate $\nu = 0.9696$ in the upper-left panel of c.2) and, condition on relative Pareto weights, employment, lender's value and transfers are monotone with shocks; furthermore, consumption risk-sharing result in borrower's values being almost the same across shocks (lower-left panel in c.1).

Figure 1(d) shows the policies of the fund contract with two-sided limited commitment. It is interesting to compare the evolution of relative Pareto weights in the upper-left panels of (d.2) and (c.2) and, correspondingly, the consumption policies in (d.1) and (c.1). With limited enforcement, the relative Pareto weight of the borrower cannot decrease indefinitely in (d.2), while it does on (c.2), but even if in principle one could assign a relatively hight weight to the borrower, limited enforcement or commitment of the lender sets upper bounds on how much Pareto weight can be given to the borrower in (d.2), while there are no bounds in (c.2) (except for an implicit one in period zero of the contract, due to the initial participation constraint of the lender). The figure also shows how these bounds depend on the shocks and how relative borrower's weights (and consumptions) decrease when limited enforcement constraints are not binding. In particular, with probability one the worst state (e_1, g_1) occurs which sets an upper bound of 0.015, given the limited enforcement constraint of the lender. After that the relative weight of the borrower can only increase with a e_9 shock, achieving 0.16 after a $e_9 g_3$ shock; therefore, in the stationary distribution of weights has a support of $[0.015, 0.16]$. The support of the stationary distribution of weights defines the relevant region of all other panels. In particular, the fact that in this economy, in contrast with the incomplete markets economy with default, there are no reversals of labor decisions (i.e. not 'working harder in bad times' as the upper-right panel of (d.1) shows); or the evolution of positive and negative transfers from the borrower to the lender (primary surpluses and deficits in the lower-left panel of d.2) or the possible *negative spreads* due to the risk of having the lender participation constraint binding (upper-left panel of d.2), which also will happen with probability one.

5.2 Computational experiments.

In this section, we discuss the simulation results from three different experiments. The first, denoted *Business Cycle Paths* (Figure 2), is a long-run simulation. In the second, denoted *Crisis Paths* (Figure 3), we assume that the economy is hit by the worst combination of shocks (e_1, g_1) and this bad state persists through the simulation. In the third, denoted *Impulse Responses* (Figure 4), we assume that the economy is hit by negative (θ, G) shocks (e_1, g_1) but then all shocks after the initial period follow a realization of the (θ, G) stochastic process; therefore we report the average impulse response from 500 simulations. The initial endogenous conditions in these experiments are set at zero assets in the incomplete markets economies and for the *FSF* economies the initial relative Pareto weight satisfies the zero value initial condition for the lender.

The three experiments have a common pattern, that sets the (**IMD** and (**2S**) economies apart. While the former tends to follow the patterns of the (**FB**) economy (subject to enforcement constraints), the latter tends to follow the patterns of the Autarkic economy. This is a pattern shared with many other simulated economies (not reported here), but specially salient to this economy where the zero assets condition, together with the possibility of default and the fact that (θ, G) are not too severe, translates in a very limited borrowing capacity in period zero, particularly when, as in our shock experiments, the initial shocks are the worst ones (e_1, g_1) (recall $D(s, b)$ in bottom-left panel of Fig. 1 (b.2)).

More specifically, Figure 2 (a) shows a typical realization of the incomplete markets economy where default takes place and before that we observe a sequence of episodes with *positive spreads*. In particular, default takes place (in period 123) after a relatively long period of a maximum level of debt, sustained by a small primary surplus, and within a short period of combined good θ and bad G shocks, as if this is 'a good time to default'; after that the economy is in autarky. The same sequence of shocks results in fairly different outcomes in the economy with a *FSF* and two-sided limited commitment (Figure 2 (c)). In particular, the implicit level of debt (negative asset holdings) is also relatively high for a relatively long period before the period in which the **IMD** economy defaults – in fact, the level of debt is much higher than in the **IMD** economy – but *FSF* allows for a combination of a higher level of primary surplus and an even higher level of debt after period 123. Around this period (particularly, just after period 123) the *FSF* economy faces the risk of the limited enforcement constraint of the lender being binding, which translates in episodes of *negative spreads*. As a result, borrower's welfare not only is higher in the *FSF* economy in period zero, but it is also particularly higher around and after the default period of the **IMD** economy. Notice, however, that consumption is higher, while labor is not, in the **IMD** economy around period 123, as if the economy fully endogenizes that it is 'a good time to default.' Figure 2 (b) compares the **2S** economy with the **1S** and the autarkic economies. The former two are almost indistinguishable except for the fact that in the **2S** economy there are episodes of *negative spreads* (i.e. with an appropriate amplification we could also detect the difference between the two economies in other panels). In particular, both *FSF* economies show smooth decreasing of consumption when the borrower's enforcement constraint is not binding and, more significantly, much more restrain (lower consumption and higher labour effort) than the autarkic economy around period 123, the period in which there is default in the **IMD** economy.

Figure 3, with the Crisis Paths, reflects the fact, already mentioned, that with persistent bad (θ, G) shocks the **IMD** economy is very close to the autarkic economy (again one needs further amplification to discriminate among these two economies) and, therefore, that in times of crisis the **IMD** economy and the economy with a *FSF* and two-sided limited commitment are further apart, for example in terms of borrower's welfare (see Fig. 3 (b)) . Figure 3 (a) shows how the *FSF* economies slowly converge to the autarkic solution. It is this transition period, in a persistent sequence of bad (θ, G) shocks, which gives the clear dominance – in terms of efficiency – to the *FSF* economies in times of crisis.

Finally, Figure 4 shows how the different economies react to a transitory combination of bad (θ, G) shocks. Again, the crisis is much worse in the incomplete markets economy with default (**IMD**) than in the economy with a *FSF* and two-sided limited commitment; the former following autarkic patterns and the latter relatively close to the first-best economy (shown in Figure 4 (a)). As Figure 4 (d) shows, consumption, labour and primary deficit patterns are very different in the **IMD** and the **2S** economies for almost twenty periods which, given the relative impatience of the borrowers, translates into substantial welfare differences in period zero. Furthermore, Figure 4 (c) shows that within economies with a *FSF* it makes a difference, in periods of crisis, whether the investor (the fund) can fully commit (**1S**) or ex-post transfers are constrained to satisfy investor's limited enforcement constraints (**2S**).

5.3 Welfare comparisons

To complete the previous analysis we now provide a quantitative welfare comparison between the economy with a *FSF* and two-sided limited commitment (**2S**) and the incomplete markets economy with default (**IMD**); in other words, a measure of the value of substituting sovereign debt financing by sovereign financing through a well designed *FSF*.

We compute a simple measure, χ, of *consumption equivalence*, taking advantage of the decomposition of the welfare functions $V_b^j = V_{b,c}^j + V_{b,n}^j$, where $j = i, f$, corresponds to the incomplete markets (**IMD**) and the *FSF* (**2S**) economies, respectively and the subscripts c and n correspond to the corresponding decomposition between consumption and labour effort. In particular,

$$V_{b,c}^j = \log\left(c^j\right) + \beta E V_{b,c}^{j\prime} = E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t^j)$$
$$V_{b,n}^j = \gamma \frac{\left(1-n^j\right)^{1-\sigma}}{1-\sigma} + \beta E V_{b,n}^{j\prime}$$

The χ measure solves the following *consumption equivalence*:

$$\begin{aligned} V_b^f &= E_0 \sum_{t=0}^{\infty} \beta^t \log((1+\chi)c_t^i) + V_{b,n}^i = \\ &= \frac{\log(1+\chi)}{1-\beta} + E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t^i) + V_{b,n}^i = \\ &= \frac{\log(1+\chi)}{1-\beta} + V_{b,c}^i + V_{b,n}^i \\ &= \frac{\log(1+\chi))}{1-\beta} + V_b^i \\ &\Rightarrow \quad (1+\chi) = \exp\left(\left(V_b^f - V_b^i\right)(1-\beta)\right). \end{aligned}$$

In other words, χ compensates in consumption the differences in consumption and labour across the two economies.The following Table 2 reports the values of χ, computed from directly from the policy functions, when initial assets are zero[7] for different initial realizations of (θ, G) shocks.

Table 2: Welfare gains under the FSF in consumption equivalent terms

Shocks (θ, G)	Debt b	Welfare Gain χ
$(\theta_l, G_h) = (0.148, 0.05)$	0	0.337
$(\theta_m, G_h) = (0.256, 0.05)$	0	0.197
$(\theta_h, G_h) = (0.444, 0.05)$	0	0.146
$(\theta_l, G_l) = (0.148, 0)$	0	0.189
$(\theta_m, G_l) = (0.256, 0)$	0	0.148
$(\theta_h, G_l) = (0.444, 0)$	0	0.119

As it can be seen In our economies the welfare gains of a *FSF contract* are very substantial (between 12% and 34% depending of having good or bad (θ, G) shocks). As it has been said, it should be taken into account that the estimates of the θ process using world TFP by Bai and Zhang (2010) results in relatively high volatile productivity shocks; with lower volatility – say, using estimates for EU countries – efficiency gains should be lower, but positive. However, it should also be taken into account that less θ volatility (or reducing G shocks or their volatility) also makes the default to autarky less costly, which results in higher default rates in the incomplete markets economy and, therefore, less borrowing capacity in this economy. As a result, as long the first best economy is more efficient than the autarkic economy, the the economy with a *FSF* and two-sided limited commitment (**2S**)dominates the incomplete markets economy with default (**IMD**), provided that the lender's participation values are not too tight.

5.4 Conclusions

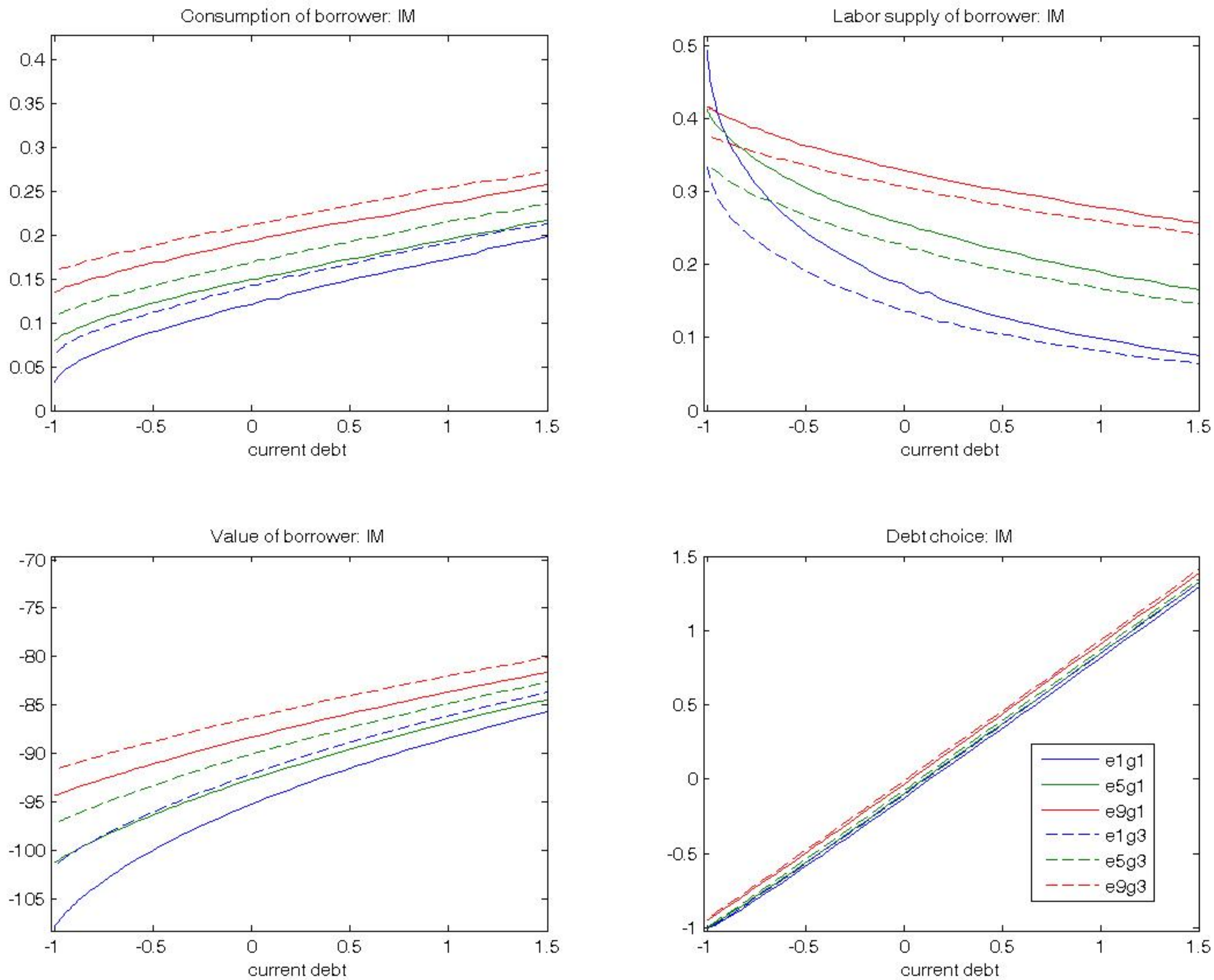
By developing and computing a model of a *Financial Stability Fund* we have provided a useful instrument to study the gains of implementing it, as well as how it should be implemented; to estimate how different sovereign debt crisis could be, and can be handled, with it. As usual, practical implementation has complexities beyond our analysis, but if anything this only underlines the need to fill the gap between the ample experience with debt financing and fund interventions and the almost inexistent theory. Part of this gap can be explained by the need to use advanced tools of dynamic contract design to properly model a *FSF*. To bring these tools to develop such model, and to contrast it with standard sovereign debt financing, we think is the main contribution of this paper. More work needs to be done, on the theoretical side, such as to account for moral hazard constraints, since the fund contract requires active conditional positive and negative transfers.

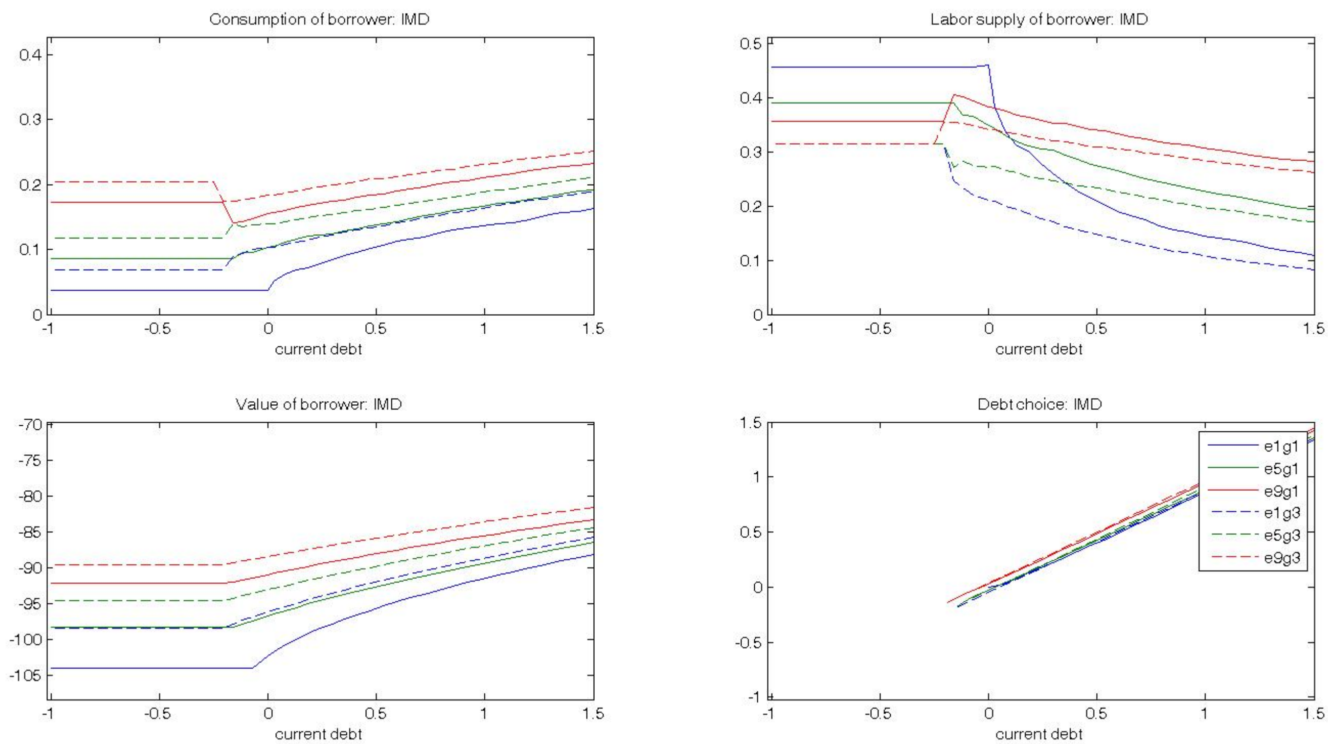
[7] In these computed economies there is almost no difference between considering initial zero assets or an initial zero value for the lender (differences in discount factors can result in differences between these two measures).

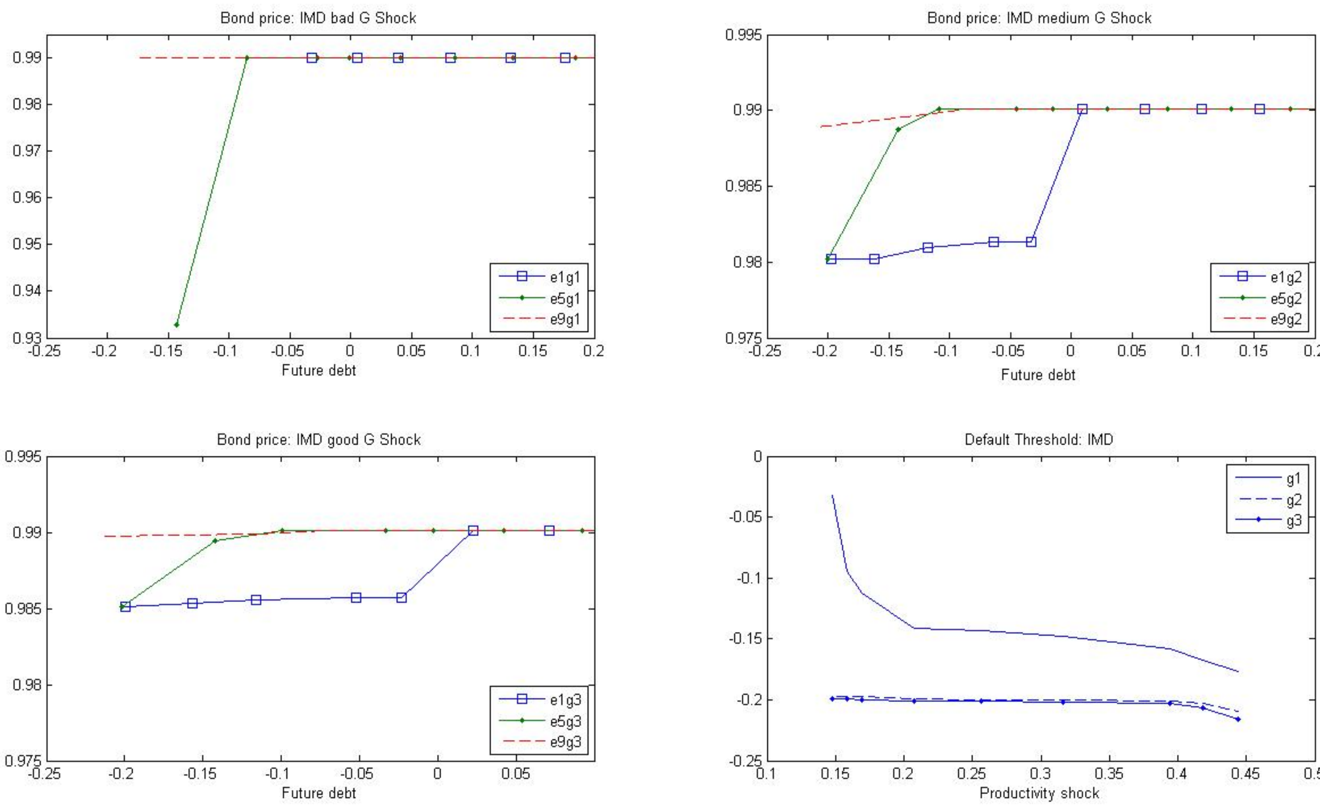
The gains that we compute for establishing a *FSF* are very large. This can partially be explained by the implied volatility of using world TFP data, following Bal and Zhang (2010). We are working on new calibrations based on a smaller number of European countries, which should also allow for a better calibration of the government shock process, as well as to provide a better estimate of the gains of having a properly designed *ESM*. In such a sample, volatility may be lower and, as a result, the gains may be lower. Nevertheless, one should also consider that we do not account for the investment and social gains of having a *FSF* in times of crisis, high positive spreads and recessions.

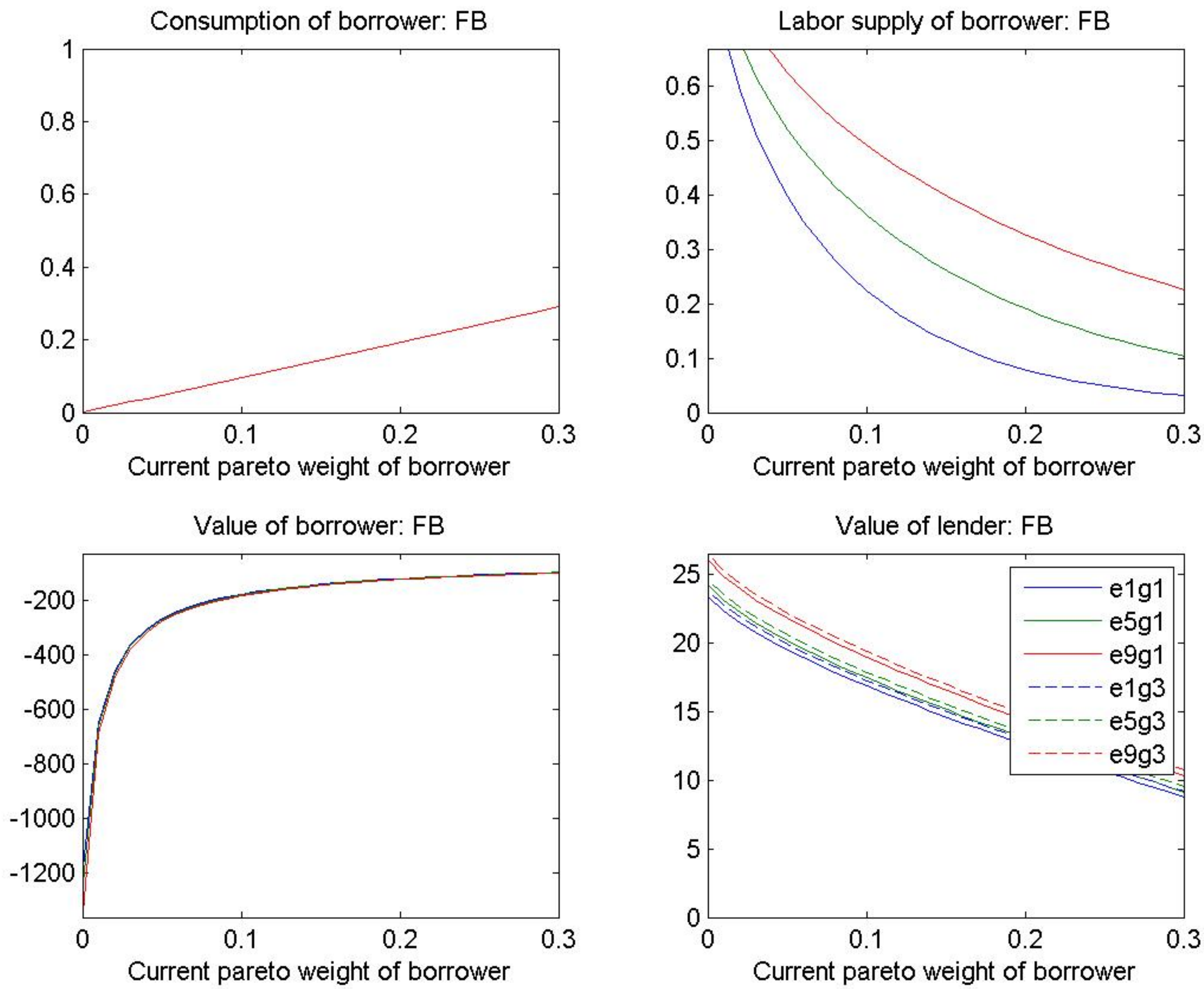
FIGURES

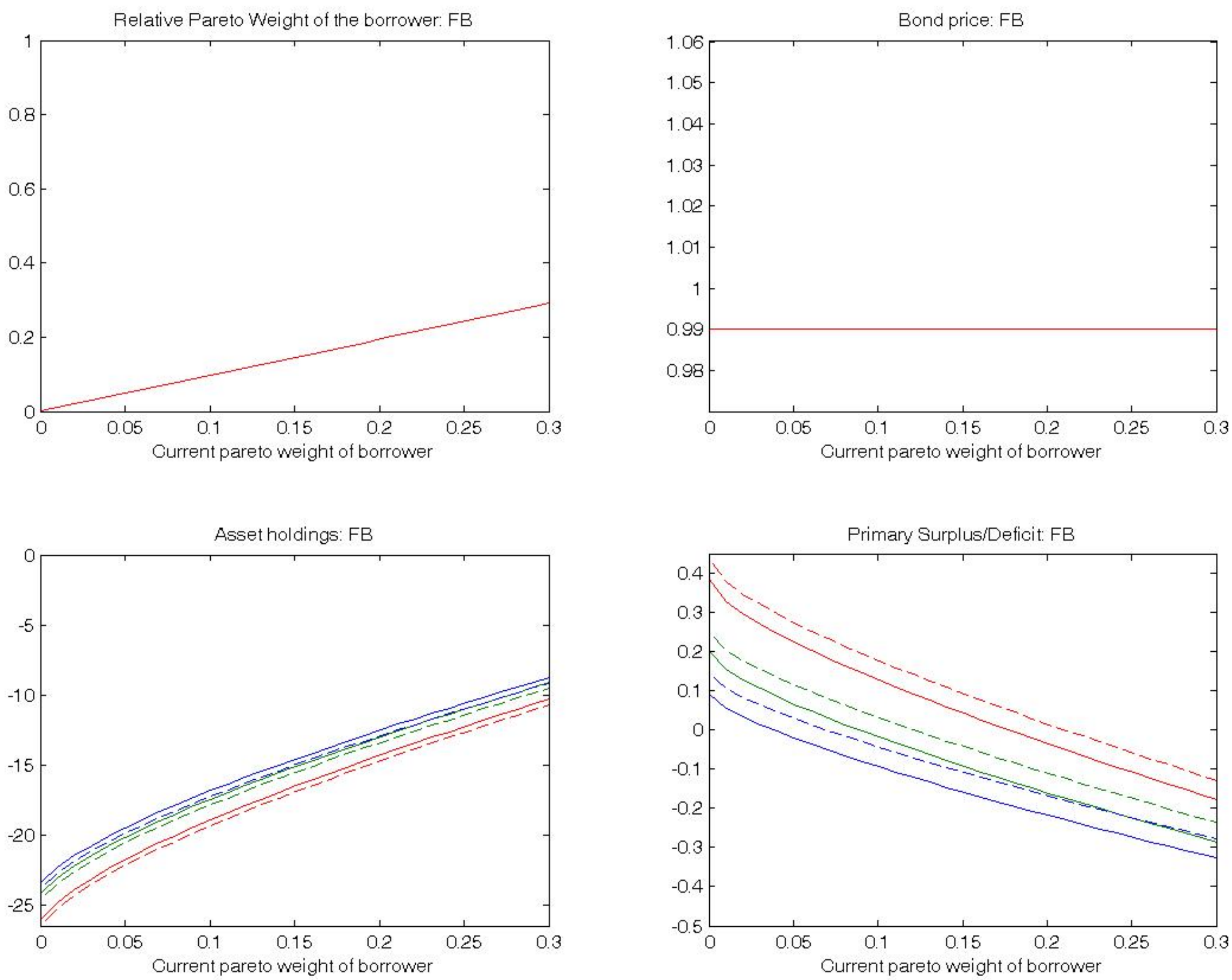
Fig. 1. Policy functions: (a) *incomplete markets* without default (IM), and (b) with default (IMD); (c) First Best (FB); (d) *FSF* with with two-sided limited commitment (2S)

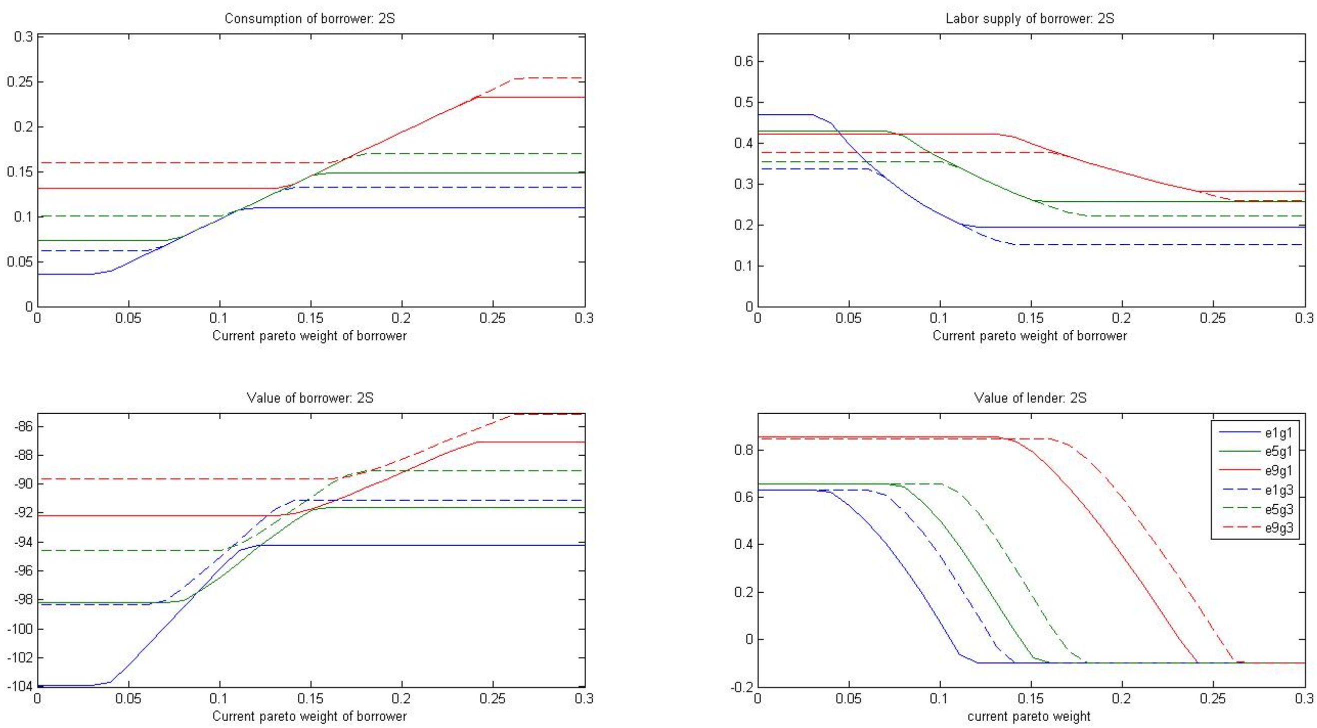

Consumption of borrower: IM
current debt
Labor supply of borrower: IM
current debt
Value of borrower: IM
current debt
Debt choice: IM
current debt
e1g1
e5g1
e9g1
e1g3
e5g3
e9g3


Consumption of borrower: IMD
current debt
Labor supply of borrower: IMD
current debt
Value of borrower: IMD
current debt
Debt choice: IMD
current debt
e1g1
e5g1
e9g1
e1g3
e5g3
e9g3


Bond price: IMD bad G Shock
e1g1
e5g1
e9g1
Future debt
Bond price: IMD medium G Shock
e1g2
e5g2
e9g2
Future debt
Bond price: IMD good G Shock
e1g3
e5g3
e9g3
Future debt
Default Threshold: IMD
g1
g2
g3
Productivity shock


Consumption of borrower: FB
Current pareto weight of borrower
Labor supply of borrower: FB
Current pareto weight of borrower
Value of borrower: FB
Current pareto weight of borrower
Value of lender: FB
e1g1
e5g1
e9g1
e1g3
e5g3
e9g3
Current pareto weight of borrower


Relative Pareto Weight of the borrower: FB
Current pareto weight of borrower
Bond price: FB
Current pareto weight of borrower
Asset holdings: FB
Current pareto weight of borrower
Primary Surplus/Deficit: FB
Current pareto weight of borrower


Consumption of borrower: 2S
Current pareto weight of borrower
Labor supply of borrower: 2S
Current pareto weight of borrower
Value of borrower: 2S
Current pareto weight of borrower
Value of lender: 2S
current pareto weight
e1g1
e5g1
e9g1
e1g3
e5g3
e9g3

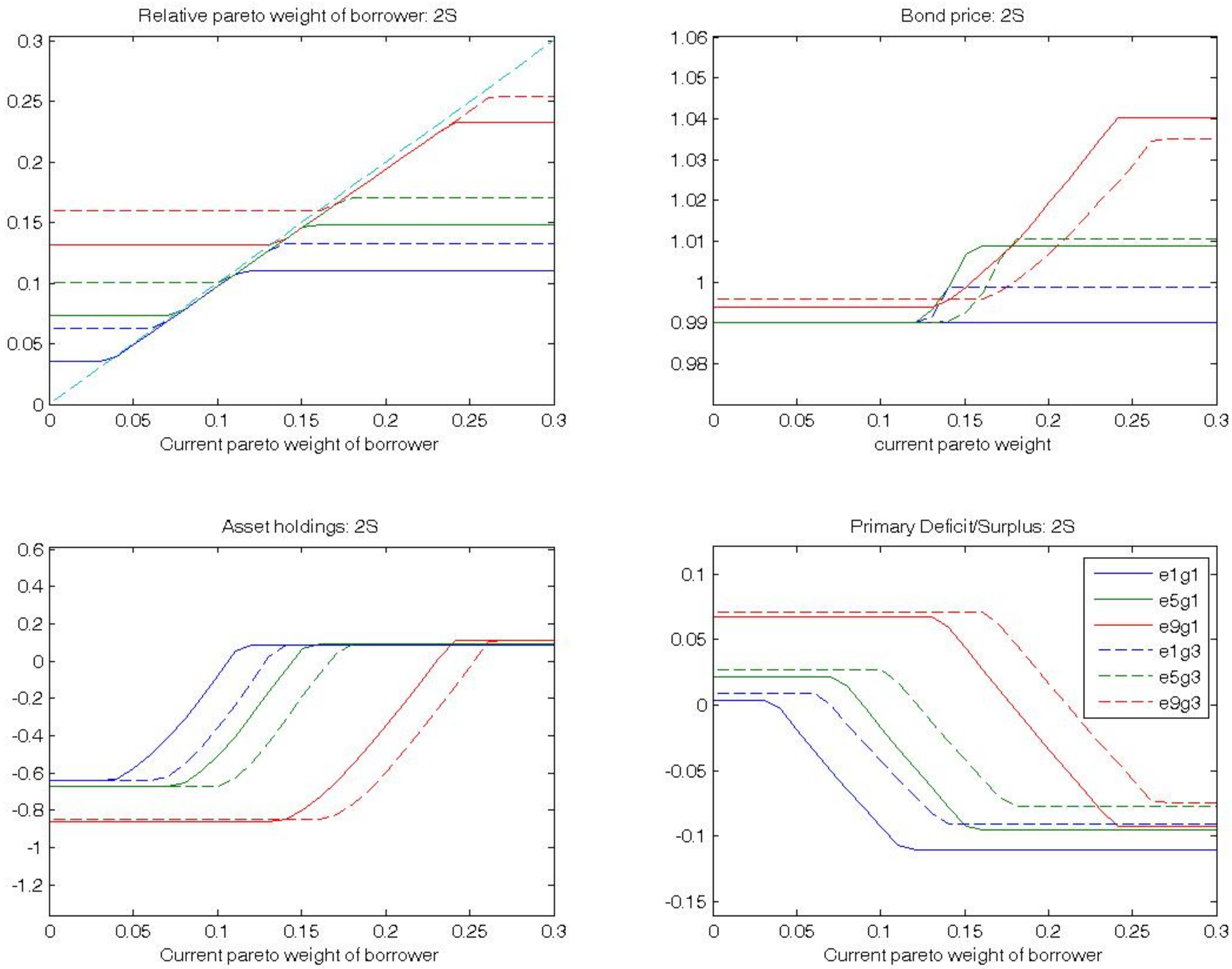
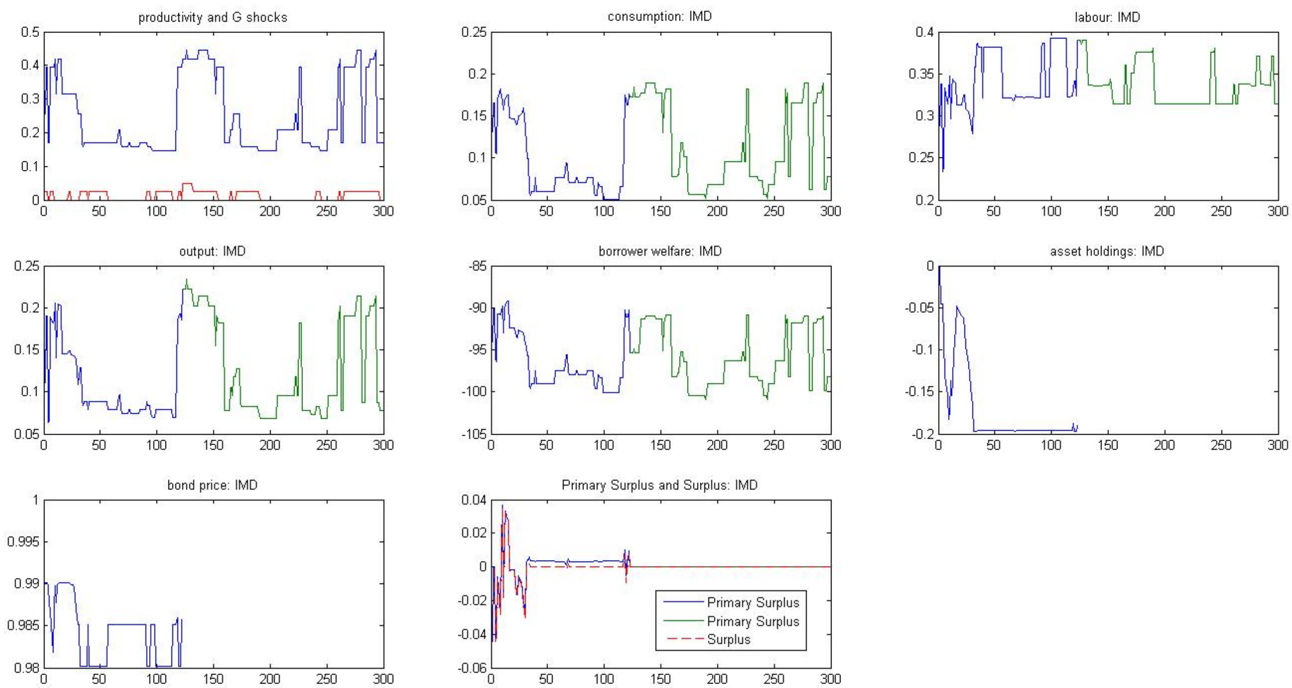
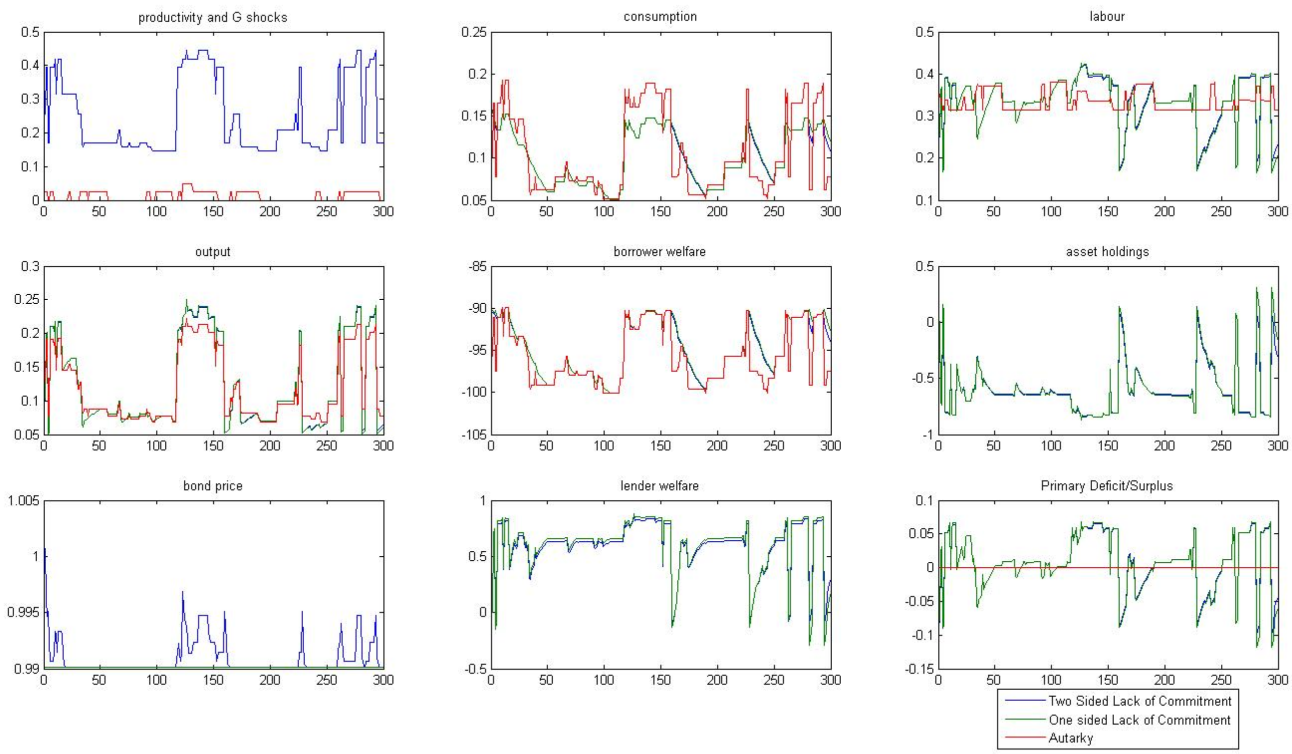
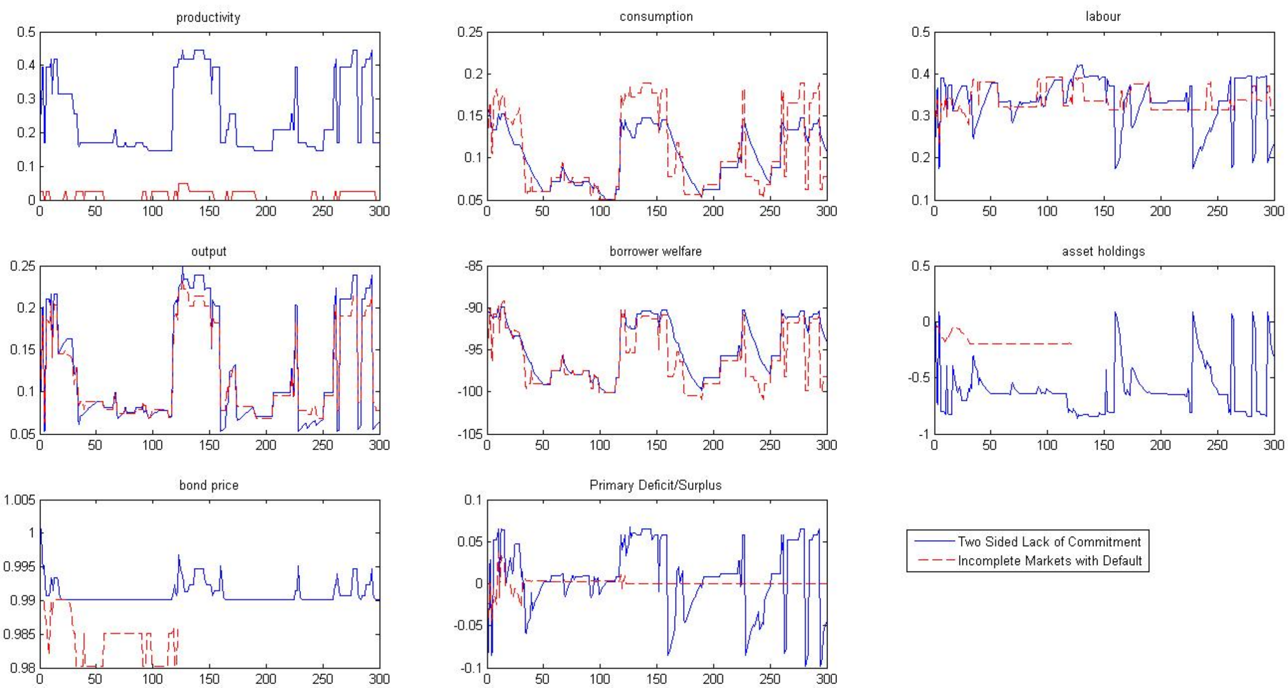

Relative pareto weight of borrower: 2S
Current pareto weight of borrower
Bond price: 2S
current pareto weight
Asset holdings: 2S
Current pareto weight of borrower
Primary Deficit/Surplus: 2S
Current pareto weight of borrower
e1g1
e5g1
e9g1
e1g3
e5g3
e9g3

Fig. 2. Business Cycle Paths: (a) *incomplete markets* with default (IMD); (b)*FSF* with one-sided (1S) and two-sided (2S) limited commitment, and Autarky; (c) two-sided limited commitment (2S) and and (IMD).

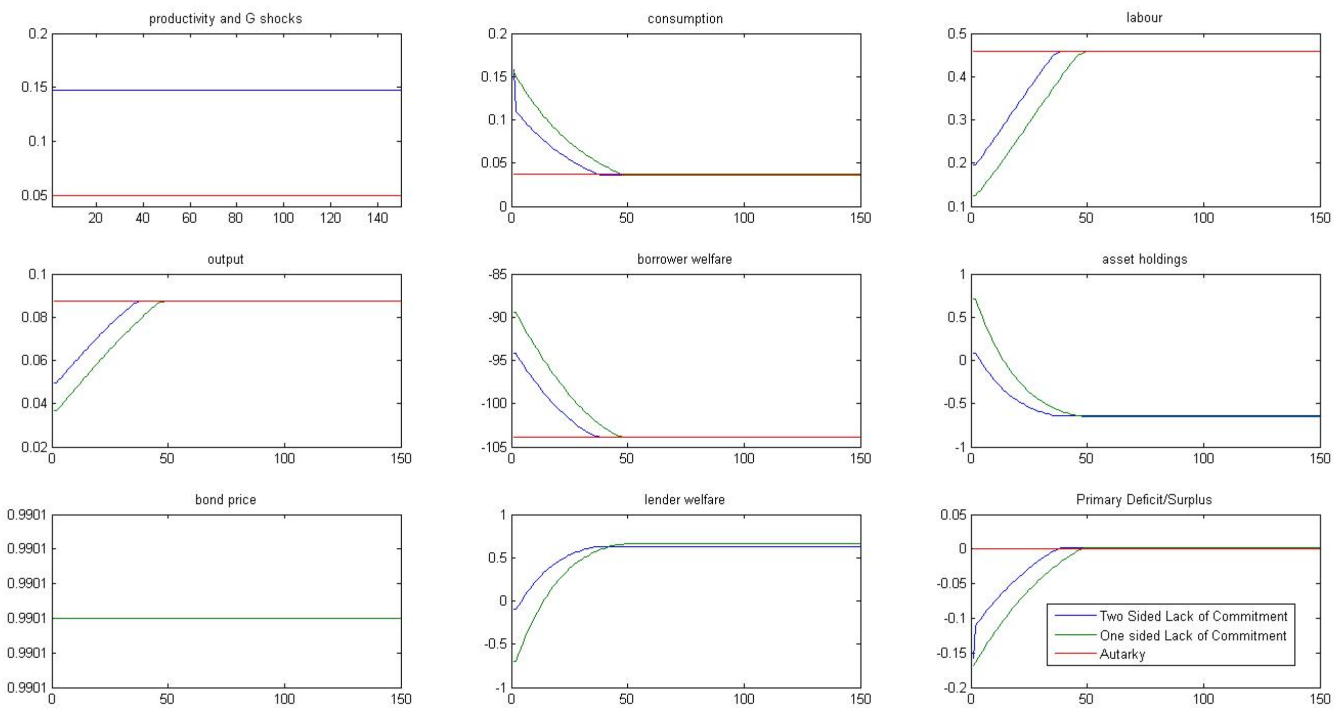

productivity and G shocks
consumption: IMD
labour: IMD
output: IMD
borrower welfare: IMD
asset holdings: IMD
bond price: IMD
Primary Surplus and Surplus: IMD
Primary Surplus
Primary Surplus
Surplus


productivity and G shocks
consumption
labour
output
borrower welfare
asset holdings
bond price
lender welfare
Primary Deficit/Surplus
Two Sided Lack of Commitment
One sided Lack of Commitment
Autarky



productivity
consumption
labour
output
borrower welfare
asset holdings
bond price
Primary Deficit/Surplus
Two Sided Lack of Commitment
Incomplete Markets with Default

Fig. 3. Crisis Paths (persistent negative (θ, G) shocks): (a) Autarky and *FSF* with one-sided (1S) and two-sided (2S) limited commitment, and (b) (2S) and (IMD).



productivity and G shocks
consumption
labour
output
borrower welfare
asset holdings
bond price
lender welfare
Primary Deficit/Surplus
Two Sided Lack of Commitment
One sided Lack of Commitment
Autarky

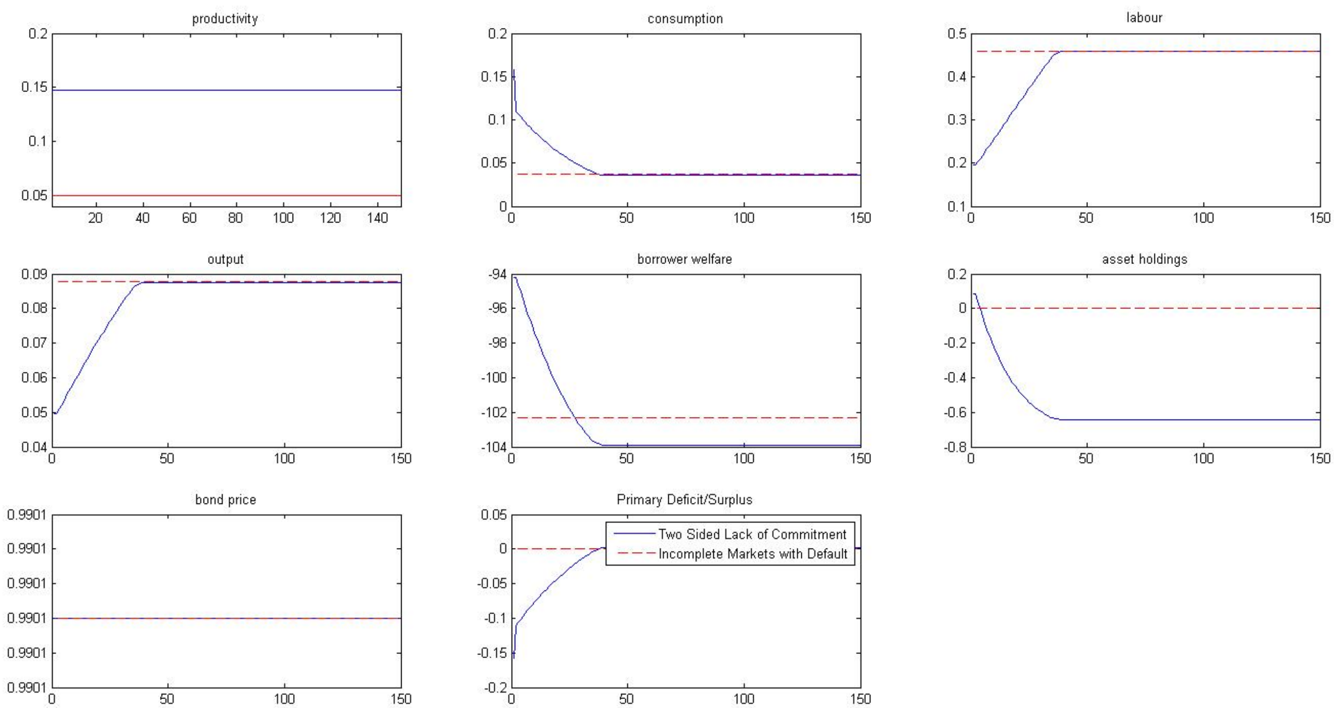
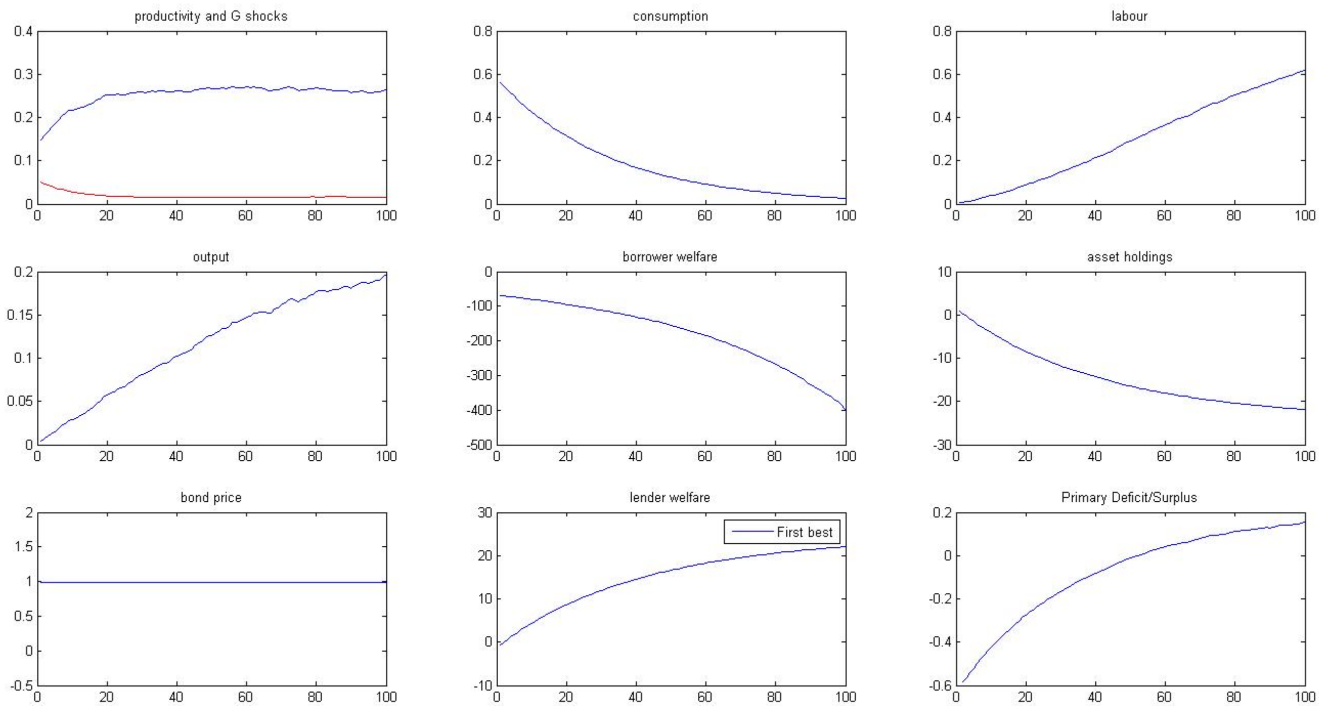
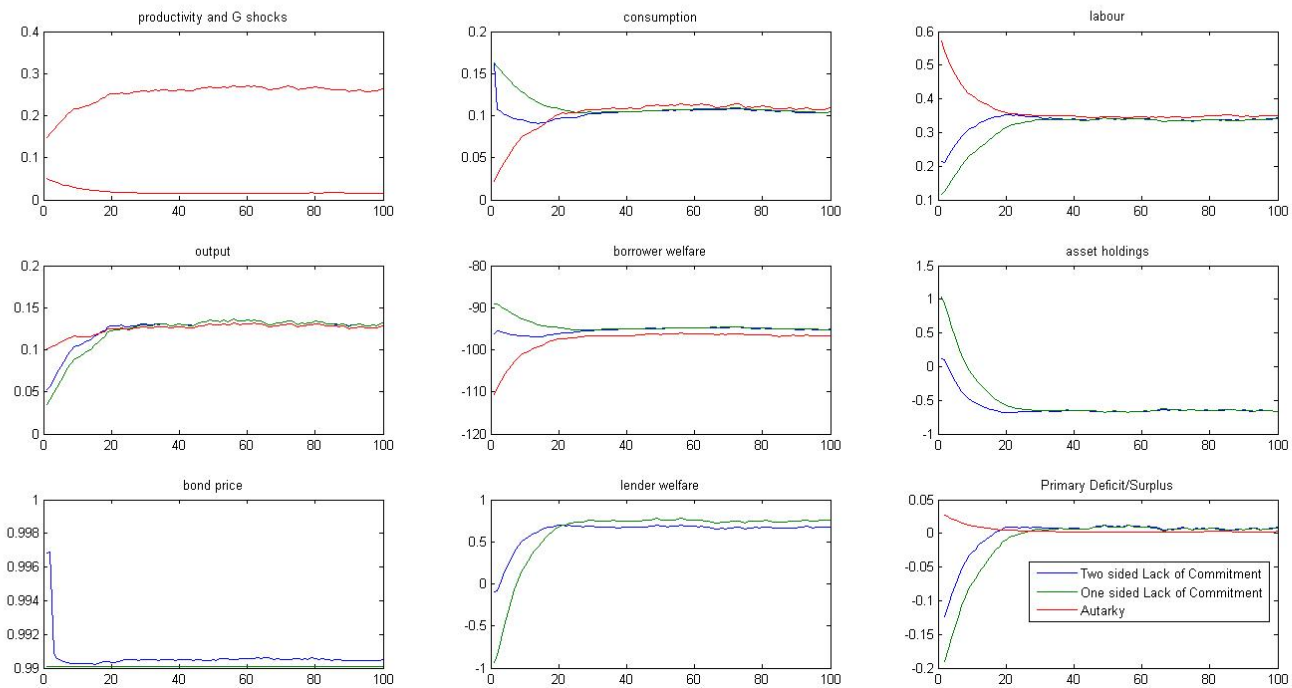
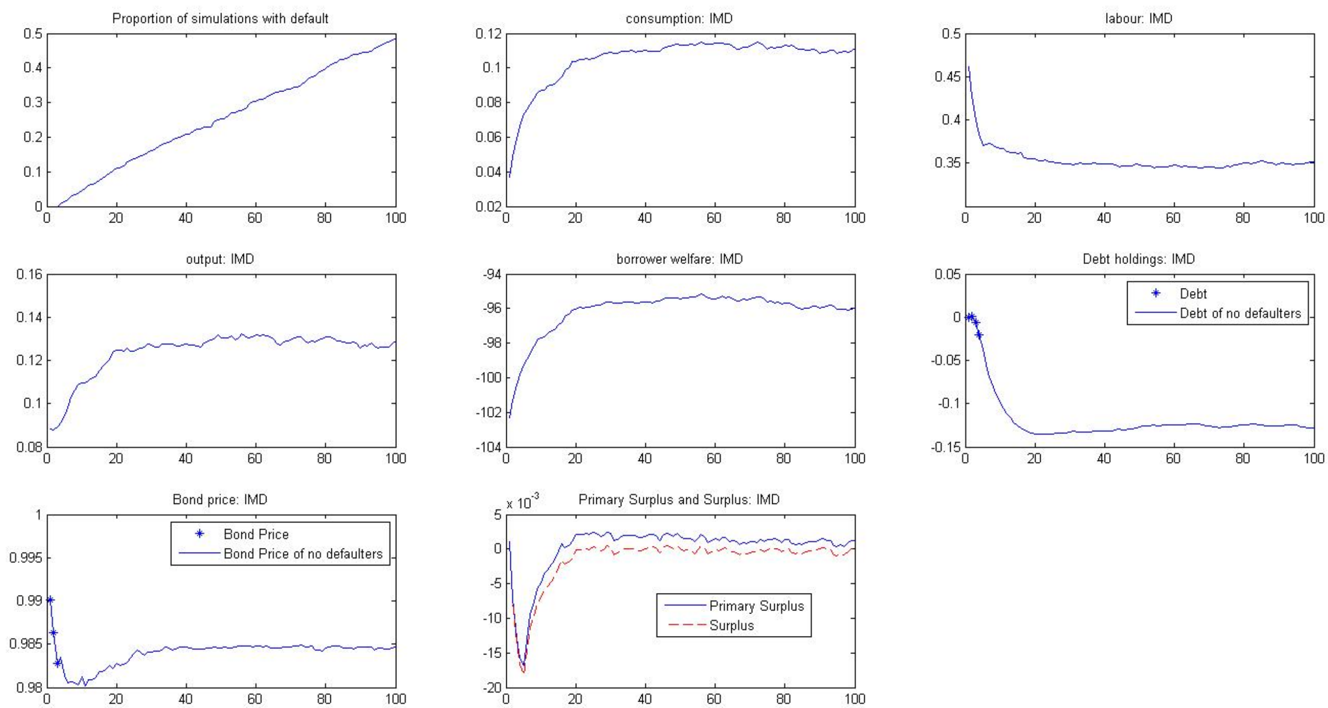

productivity
consumption
labour
output
borrower welfare
asset holdings
bond price
Primary Deficit/Surplus
Two Sided Lack of Commitment
Incomplete Markets with Default

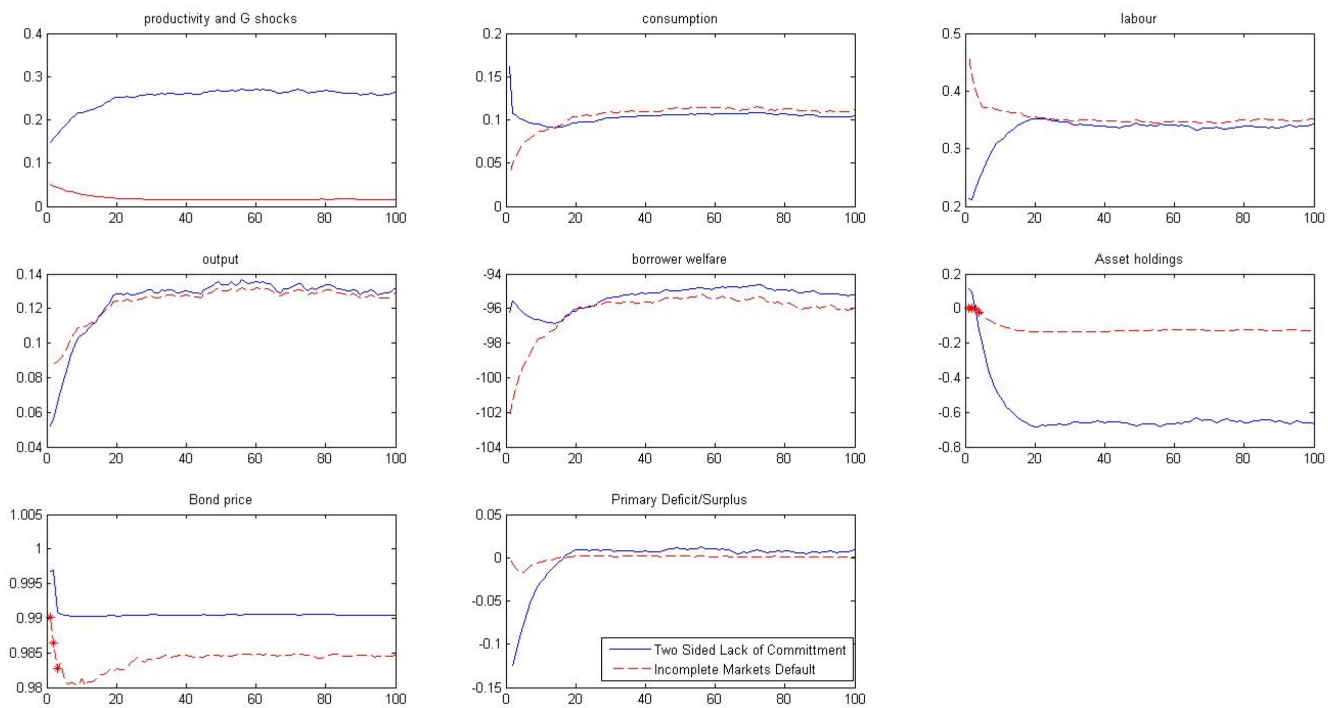
Fig. 4. Reactions to a shock (average Impulse response to negative (θ, G) shocks:(a) First Best (FB); (b) Autarky and *FSF* with one-sided (1S) and two-sided (2S) limited commitment; (c) *incomplete markets* with default (IMD) and (d) (2S) and (IMD).



productivity and G shocks
consumption
labour
output
borrower welfare
asset holdings
bond price
lender welfare
First best
Primary Deficit/Surplus


productivity and G shocks
consumption
labour
output
borrower welfare
asset holdings
bond price
lender welfare
Primary Deficit/Surplus
Two sided Lack of Commitment
One sided Lack of Commitment
Autarky


Proportion of simulations with default
consumption: IMD
labour: IMD
output: IMD
borrower welfare: IMD
Debt holdings: IMD
Debt
Debt of no defaulters
Bond price: IMD
Bond Price
Bond Price of no defaulters
Primary Surplus and Surplus: IMD
Primary Surplus
Surplus


productivity and G shocks
consumption
labour
output
borrower welfare
Asset holdings
Bond price
Primary Deficit/Surplus
Two Sided Lack of Committment
Incomplete Markets Default

6 Appendix

6.1 Asset prices and transverslty conditions

We first show that asset prices can be defined by (8).To see this, notice that the optimality conditions of the fund contract imply:

$$\frac{\beta u'(c_b(s^t))}{u'(c_b(s^{t-1}))} = \frac{1}{(1+r)}\frac{(1+v_l(s^t))}{(1+v_b(s^t))}$$

If the participation constraint is not binding for either agent then $v_b\left(s^t\right) = v_l\left(s^t\right) = 0$ and

$$\frac{\beta u'(c_b(s^t))}{u'(c_b(s^{t-1}))} = \frac{1}{(1+r)}$$

If the participation constraint is binding only for the lender, then $v_b\left(s^t\right) = 0$ and $v_l\left(s^t\right) > 0$ so that:

$$\frac{\beta u'(c_b(s^t))}{u'(c_b(s^{t-1}))} = \frac{1}{(1+r)}\left(1+v_l(s^t)\right) > \frac{1}{(1+r)}$$

If the participation constraint is binding only for the borrower then $v_b\left(s^t\right) > 0$ and $v_l\left(s^t\right) = 0$ so that:

$$\frac{\beta u'(c_b(s^t))}{u'(c_b(s^{t-1}))} = \frac{1}{(1+r)}\frac{1}{(1+v_b(s^t))} < \frac{1}{(1+r)}$$

It follows that (8) properly defines the price of the one period state contingent claim.

We now show that the transversality conditions in the competitive equilibrium are satisfied:

$$\begin{aligned}
& \lim_{t\to\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)u'\left(c_b^*\left(s^t\right)\right)\left[a_b\left(s^t\right)-A_b\left(s^t\right)\right] \\
\leq\; & \lim_{t\to\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)u'\left(c_b^*\left(s^t\right)\right)\left[\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t}Q\left(s^{t+n}|s^t\right)\left[c_b^*\left(s^{t+n}\right)\right]\right] \\
\leq\; & u'\left(c_b^*\left(s_0\right)\right)\lim_{t\to\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)\frac{u'\left(c_b^*\left(s^t\right)\right)}{u'\left(c_b^*\left(s_0\right)\right)}\left[\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t}Q\left(s^{t+n}|s^t\right)\left[c_b^*\left(s^{t+n}\right)+c_l^*\left(s^{t+n}\right)\right]\right] \\
\leq\; & u'\left(c_b^*\left(s_0\right)\right)\lim_{t\to\infty}\sum_{s^t}Q\left(s^t|s_0\right)\left[\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t}Q\left(s^{t+n}|s^t\right)\left[c_b^*\left(s^{t+n}\right)+c_l^*\left(s^{t+n}\right)\right]\right] = 0
\end{aligned}$$

and

$$
\begin{aligned}
& \lim_{t\to\infty}\sum_{s^t}\left(\frac{1}{1+r}\right)^t \pi\left(s^t\right)\left[a_l\left(s^t\right)-A_l\left(s^t\right)\right] \\
\leq\ & \lim_{t\to\infty}\sum_{s^t}\left(\frac{1}{1+r}\right)^t \pi\left(s^t\right)\left[\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q\left(s^{t+n}|s^t\right)\left[c_l^*\left(s^{t+n}\right)\right]\right] \\
\leq\ & \lim_{t\to\infty}\sum_{s^t}\left(\frac{1}{1+r}\right)^t \pi\left(s^t\right)\left[\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q\left(s^{t+n}|s^t\right)\left[c_b^*\left(s^{t+n}\right)+c_l^*\left(s^{t+n}\right)\right]\right] \\
\leq\ & \lim_{t\to\infty}\sum_{s^t} Q\left(s^t|s_0\right)\left[\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q\left(s^{t+n}|s^t\right)\left[c_b^*\left(s^{t+n}\right)+c_l^*\left(s^{t+n}\right)\right]\right]=0
\end{aligned}
$$

The first inequalities follow from the definitions of $a\left(s^t\right)$ and $A\left(s^t\right)$. The second inequalities follow from the fact that individual consumption is less than aggregate consumption. The third inequalities follow from the relationship between q and Q as well as the relationship between q and the marginal rates of substitution of the agents. The last inequalities follow from the assumption of high implied interest rates.

6.2 Solution Method

6.2.1 Long term contracts

To solve the model, we assume the following utility for the borrower: $\log(c)+\frac{\gamma(1-n)^{1-\sigma}}{1-\sigma}$and the following production function θn^{α}. With these functional forms, the equilibrium conditions are:

$$\frac{1}{c\left(x,s\right)}=\frac{1+\upsilon_l(x,s)}{1+\upsilon_b(x,s)}\frac{x}{\eta}=x'$$

$$c\left(x,s\right)\gamma\left(1-n\left(x,s\right)\right)^{-\sigma}=\theta\alpha n\left(x,s\right)^{\alpha-1}$$

$$V^{bf}(x,s)=\log(c(x,s))+\frac{\gamma(1-n(x,s))^{1-\sigma}}{1-\sigma}+\beta\sum_{s'\in S}\pi(s'|s)V^{bf}(x',x').$$

$$V^{lf}(x,s)=\theta n(x,s)^{\alpha}-c(x,s)+\frac{1}{1+r}\sum_{s'\in S}\pi(s'|s)V^{lf}(x',s').$$

$$V^{af}(s)=\max_n\left\{\begin{array}{c}\log(\theta n^{\alpha})+\frac{\gamma(1-n)^{1-\sigma}}{1-\sigma}+\beta\sum_{s'\in S}\pi(s'|s)\left(1-\lambda\right)V^{af}(s') \\ +\beta\sum_{s'\in S}\pi(s'|s)\lambda V^{bf}(x^*(s'),s')\end{array}\right\}$$

For simplicity, we rewrite the model differently using as a state variable the relative pareto weight for the borrower $z = \frac{1}{x}$, the system of equations above can then be rewriten as:

$$\begin{aligned} \frac{1}{c(z,\theta)} &= \frac{1+\upsilon_l(z,\theta)}{1+\upsilon_b(z,\theta)} \frac{1}{\eta z} = \frac{1}{z'} \\ z' &= \frac{1+\upsilon_b(z,\theta)}{1+\upsilon_l(z,\theta)} \eta z \end{aligned}$$

$$c(z,\theta)\gamma(1-n(z,\theta))^{-\sigma} = \theta\alpha n(z,\theta)^{\alpha-1}$$

$$V^{bf}(z,\theta) = \log(c(z,\theta)) + \frac{\gamma(1-n(z,\theta))^{1-\sigma}}{1-\sigma} + \beta\sum_{\theta'\in S}\pi(\theta'|\theta)V^{bf}(z',\theta').$$

$$V^{lf}(z,\theta) = \theta n(z,\theta)^{\alpha} - c(z,\theta) + \frac{1}{1+r}\sum_{\theta'\in S}\pi(\theta'|\theta)V^{lf}(z',\theta').$$

$$V^{af}(\theta) = \max_{n}\left\{\begin{array}{c} \log(\theta n^{\alpha}) + \frac{\gamma(1-n)^{1-\sigma}}{1-\sigma} + \beta\sum_{\theta'\in S}\pi(\theta'|\theta)(1-\lambda)V^{af}(\theta') \\ +\beta\sum_{\theta'\in S}\pi(\theta'|\theta)\lambda V^{lf}(z^*(\theta'),\theta') \end{array}\right\}$$

To solve the problem, we use a policy function iteration algorithm that we describe in what follows:

- We discretize the shock for the borrower θ (S values) and the relative pareto weight for the borrower z (N values).
- For each grid point, we can calculate the value of autarky as follows:
  - find the optimal labor in autarky, which solves $\frac{\alpha}{n} = \gamma(1-n)^{-\sigma}$.
  - find $V^{bf}(z^*(\theta'),\theta')$ by first solving for $z^*(\theta')$ such that $V^{lf}(z^*(\theta'),\theta') = 0$.
  - calculate $V^{af}(\theta)$ from the previous equation
- We then define the region of pareto weights between which none of the participation constraints are binding as $[z(j+1), z(l-1)]$. In that region, the solution is characterized by the first best:

$$\begin{aligned} \upsilon_b(z,\theta) &= \upsilon_l(z,\theta) = 0 \\ z' &= \eta z \\ c(z,\theta) &= \eta z \text{ and } c_l(z,\theta) = \theta n(z,\theta)^{\alpha} - \eta z \\ \eta z\gamma(1-n(z,\theta))^{-\sigma} &= \theta\alpha n(z,\theta)^{\alpha-1} \\ V^{lf}(z,\theta) &= \theta n(z,\theta)^{\alpha} - \eta z + \frac{1}{1+r}\sum_{\theta'\in S}\pi(\theta'|\theta)V^{lf}(z',\theta') \\ V^{bf}(z,\theta) &= \log(\eta z) + \frac{\gamma(1-n(z,\theta))^{1-\sigma}}{1-\sigma} + \beta\sum_{\theta'\in S}\pi(\theta'|\theta)V^{bf}(z',\theta'). \end{aligned}$$

- For a given shock θ, if the value function for the borrower at the worst possible pareto weight $z(1)$ is higher than his autarky value, then his participation constraint is never binding and we set $j = 0$.

- Similarly, for a given shock θ, if the value function for the lender at the worst possible pareto weight $z(N)$ is higher than his outside option then his participation constraint is never binding and we set $l = N + 1$.

- To find the region for which the participation constraint binds for the borrower, for each shock θ, we find $c(z_b, \theta) = \eta x_b$ such that $V^{bf}(z_b, \theta) = V^{af}(\theta)$.

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