Credit Bubbles and Misallocation

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Abstract

Why are bubbles harmful to the economy? According to a recent literature on rational bubbles, the problem is the burst that causes a misallocation of funds and triggers a recession. This paper proposes a theory of rational bubbles where the boom, not the ensuing bust, reduces the output by promoting a misallocation of factors. As in recent literature, financial markets are imperfect and the rise of a bubble alleviates credit constraints and boosts capital accumulation. However, capital accumulation occurs in unproductive sectors and aggregate output is reduced. The result is driven by the fact that heterogeneous borrowers have an advantage with respect to issuing different types of debt contracts. In normal times, High-productive borrowers have higher collateral and thereby attract most of the funds. In bubbly times, borrowers can also issue "bubbly debt," a debt that is repaid with future debt. The possibility to keep a pyramid scheme and raise bubbly debt depends on the probability of surviving in the market. Therefore, a bubble misallocates resources towards borrowers with low *fundamental* risk, even if they invest in projects with lower productivity. An augmented version of the model with nominal rigidities is proposed to explain the timing of expansions and recessions during "bubbly episodes": the initial boom in output is caused by a positive demand effect; the long run reduction in TFP is driven by a misallocation process. The theory is supported by evidence on between-industry misallocation in the years preceding the 2008 financial crisis.

JEL classification: E32, E44, O16

Keywords: bubbles, credit, misallocation, financial frictions, productivity

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1 Introduction

In recent decades modern economies have experienced large fluctuations in aggregate credit. Periods of high growth have typically been followed by periods of decline or sudden busts. What drives these cycles is a current subject of research and no consensus has yet been reached. A growing literature links these periods of extraordinary credit growth to the emergence of a bubble. In particular, recent papers on rational bubbles point to the role of asset bubbles in easing the transfer of funds when credit is constrained. According to these sources, bubbles boost the productive efficiency of the economy by improving the allocation of financing - the burst of the bubble initiates a recession. However, there exists an alternative view proposing that credit booms and bubbles actually induce a direct misallocation of resources in the economy.

This paper contributes to the debate in two ways. First, it provides evidence that favors the misallocation view by analyzing the between-industry allocation of factors across Western countries in the years prior to the 2008 financial crisis. Second, it builds on recent theories put forward in the literature on rational bubbles to support this alternative hypothesis. I propose that it is the emergence of a bubble that reduces the output by promoting a misallocation of resources.

The original theory of rational bubbles was introduced by Tirole (1985). In Tirole's framework, a bubble, defined as an asset with a zero market fundamental, can appear when the economy is dynamically inefficient; i.e., when the marginal return on capital is smaller than the growth rate of the economy. Bubbles, then, enhance the inter-temporal allocation of resources and reduce the stock of capital. However, dynamic inefficiency was considered empirically irrelevant by most economists at the time.[1] In addition, real bubbly episodes are typically characterized by a boom in capital accumulation, a phenomenon that is counterfactual to the capital crowding-out predicted by the model. Recent papers relax the condition for the existence of rational bubbles and relate the arrival and burst of a bubble to credit dynamics. In fact, market returns can be lower than the growth rate even if the economy is dynamically efficient once we allow for imperfections in financial markets.[2] According to Kocherlakota (2009), Martin and Ventura (2012, 2016), and Miao and Wang (2012) a bubble improves the intra-temporal allocation of funds, from unproductive agents to credit-constrained productive ones.[3] Intuitively, a bubble in the asset market raises the value of collateral, relaxes the borrowing constraint, and therefore increases the amount of credit in the economy.[4] In these models the positive reallocation of investment supports a crowding-in of capital.

[1]See Abel, Mankiw, Summers and Zeckhauser (1989) and Geerolf (2013) for an empirical investigation on dynamic inefficiency.

[2]Woodford (1990) had already shown that financial frictions could relax the conditions for rational bubbles.

[3]Kocherlakota (2009) and Miao and Wang (2012) present models with infinite lived agents facing productivity shocks. Martin and Ventura (2012, 2016) rely on an Over-Lapping Generations model with generations of productive and unproductive agents.

[4]In Kocherlakota (2009), Miao and Wang (2012) and Martin and Ventura (2016) the bubble is on the assets playing the role of collateral. The main specification in Martin and Ventura (2012) has no explicit collateral constraint but the model can be reinterpreted with this constraint.

These recent papers on rational bubbles can replicate aggregate macroeconomic facts, such as the rise in investment rate during a credit boom and the start of a recession at the bust. Nonetheless, I question the reallocation channel which drives their result. My theory suggests that a bubble still alleviates credit constraints and raises the stock of capital. However, this is in favor of low productivity sectors.[5]

In Section 2, I provide the evidence that motivates my model. I investigate the relationship between credit growth and factor allocation in the years preceding the 2008 financial crisis. Specifically, I compare the change in between-industry allocation for a sample of Western countries that experienced a differential growth in credit. The result is that larger credit booms favored the expansion of industries with low Total Factor Productivity growth. In particular, companies from less productive industries relatively increased their leverage in the countries with a higher credit growth. In the following sections I place these facts inside the rational bubble framework.[6]

The theoretical contribution of the paper is presented in two steps, described in Sections 3 and 4. First, in a stylized model I derive the necessary conditions for bubbles inducing a misallocation of factors. Second, in a richer model I introduce a motivation for bubbles appearing and boosting capital accumulation in low productivity sectors.

My setup is based on the classical Over-Lapping Generations framework. In the model there are two types of agents: workers and investors. Workers earn their wage when young but have no technology to store their income for consumption when old. Investors, on the other hand, can invest today in order to obtain working capital tomorrow. A borrowing constraint limits the credit between workers and investors. However, the latter can potentially expand the funds they raise by issuing bubbly debt, a debt that will not be repaid with future income but with the purchase of this debt by a new generation of workers. It is worth noting that the emergence of bubbly debt is subject to workers' beliefs regarding future repayment.

A main feature in the model is heterogeneity in investor productivity. In Section 3, agents' beliefs will not only determine the rise of bubbly debt but also the identity of the issuers. Notably, the ability to issue bubbly debt does not depend on the productivity of an investor, since he will not be responsible for repayment. If workers buy bubbly debt issued by low productive investors, the outcome is a misallocation of resources away from more productive investors.

The mechanism described in Section 3 illustrates how a credit bubble can drag the economy into an

[5] To my knowledge, factor misallocation in a rational bubble environment has only been discussed in Miao and Wang (2014). According to them a bubble can arise in a specific sector. However, a sector-specific bubble does not produce any direct misallocation. In keeping with the rest of the literature, the bubble still increases the productive efficiency of the sector. The overall productivity of the economy is negatively affected because the specific sector produces a negative externality on the rest of the economy.

[6] There are alternative theories that link credit booms and misallocation. For example Cecchetti and Kharroubi (2015) show that an expansion of the financial sector misallocates high-skilled workers from more productive sectors. Alternatively, Gopinath, Kalemli-Ozcan, Karabarbounis and Villegas-Sanchez (2015) describe an environment in which larger firms have an advantage in accessing credit. However, neither paper takes into account the boom-and-bust nature of credit cycles.

inefficient allocation of factors. There are, however, two drawbacks to this model. First, it does not explain how the borrowers issuing bubbly debt are selected. Second, it predicts a reduction in aggregate capital when factors are misallocated. This prediction is counterfactual to the large accumulation of capital that preceded the 2008 financial crisis.

In Section 4, I address both issues by making a substantial addition to my model. I assume that the possibility of sustaining a bubbly scheme is subject to the survival of the issuer on the market: when a singular investor leaves the market, his bubbly debt must burst. In this section, bubbly debt is effectively repaid with future debt until such a time that a borrowing investor dies or fails.[7] In the real world, long-lived investors may be intermediaries that finance traditional sectors such as housing and real estate, activities with typically low productivities that, nonetheless, have low *fundamental* risk. Assuming that low productive investors also face a lower risk of leaving the market, they have a higher chance of issuing bubbly debt.[8] In addition, their longer life expectancy allows them to accumulate more capital over time. This implies that a bubble can boost aggregate capital even if resources are misallocated and the economy is contracting.

A crucial aspect of both versions of my model is the possibility of initiating a new bubbly scheme by issuing bubbly debt. This possibility is also included in the framework set out by Martin and Ventura (2012) where the agent who issues a bubbly asset effectively earns a rent. The authors identify two types of bubbly episodes: in contractionary episodes capital is crowded-out as in Tirole's framework; in expansionary episodes capital is crowded-in. My paper proposes a third type of bubbly episodes: capital is crowded-in while output is reduced.

In the paper I also advance an alternative hypothesis for the observed short-term positive correlation between credit, asset prices and output stressed by the recent literature on rational bubbles. In an augmented version of the model I introduce nominal rigidities and show how positive shocks to nominal returns initially provoke a positive demand effect, while they can trigger the rise of a pyramid scheme and a misallocation of factors afterwards. In particular, this modified framework can explain the long-run negative relation between aggregate productivity and credit during the periods of low inflation that advanced economies have experienced in the last decades.

The model I propose presents distinct policy implications when compared to the recent literature. Since bubbles are contractionary, a benevolent social planner would generally work to prevent their appearance.[9] A regulatory authority should limit the creation of debt by the private sector in order to manage the emergence of bubbles.

[7] A bubble here can be interpreted as a Ponzi-scheme.

[8] A low fundamental risk, clearly, does not imply an overall low risk. Interestingly, the framework predicts a negative relation between fundamental and non-fundamental risk.

[9] More exactly, depending on the parameters of the model, a social planner may favor a small bubbly scheme in order to increase aggregate consumption - even if that reduces the total output.

Besides the rational bubble literature, this paper is related to the wider literature on credit cycles and financial crisis. Empirical works by Borio and Drehmann (2009), Reinhart and Rogoff (2011), and Schularick and Taylor (2012) recognize that credit growth is a main predictor for financial crises. More recently, additional papers have addressed the effect of credit booms on factor allocation. Gopinath, Kalemli-Ozcan, Karabarbounis and Villegas-Sanchez (2015) illustrate how the allocation of capital in Spain deteriorated during the period of rapid inflows following the introduction of the euro in 1999; alternatively Borio, Kharroubi, Upper and Zampolli (2016) present a decomposition of labor productivity across Western economies and claim that credit booms provoke a misallocation of the labor force.[10] My theory is also related to the over-accumulation view of crises.[11] Note that, in the model described here, a recession does not originate from an over-accumulation of capital, but rather from an accumulation in the wrong sector.

Finally, the paper is linked to the empirical and theoretical research on liquid debt. Indeed, our bubbly debt can be naturally interpreted as a short-term or liquid bank note. Growth in aggregate credit is associated with a near-symmetric increase in bank debt. For example, Krishnamurthy and Vissing-Jorgensen (2015) describe the relation between loans and liquid debt on the two sides of the balance sheets for the US financial sector. From a theoretical perspective, our bubbly debt has similarities to the information-insensitive bank debt described by Dang, Gorton, Hölmstrom, and Ordoñez (2016) where repayment does not depend on the borrower's productivity. However, in the model set out here there is no liquidity mismatch between the assets and the liabilities of a borrower.

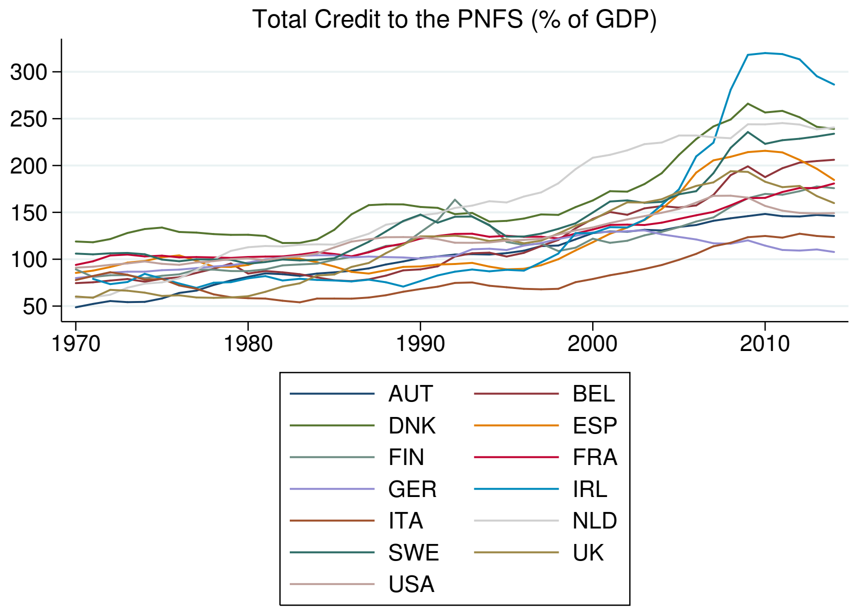
The remainder of the paper is organized as follows: Section 2 presents the empirical results that inform the theory. In Section 3, I describe the stylized version of the model in which workers' beliefs determine who can issue bubbly debt. In Section 4, I add a risk component to the activity of investors which influences their survival on the market. Here low risk investors have an advantage in the issuing of bubbly debt. Section 4 also describes the dynamics of the model with and without nominal rigidities and the policy implications of the model. Section 5 concludes.

2 Credit Booms and Between-Industry Misallocation

I motivate my theory on the basis of evidence on the allocation of factors across industries in the US and western Europe prior to the 2008 financial crisis. In Figure 1, I show the path of total credit to the Private Non-Financial Sector (PNFS) normalized by GDP. As we can see, from the late 1990s to 2008, the majority of sample countries experienced an unprecedented credit rise. For some countries, this boom was particularly

[10] The first paper focus on within-industry misallocation, while the second one looks at between-industry misallocation.
[11] Friedrich Hayek was the most notable proponent of this view on recessions.

Figure 1



Notes: Data are from the "Total credit to the non-financial sector" database by the Bank for International Settlements.

dramatic: in Ireland the credit ratio rose from 100% at the end of the 1990s to over 300% at the peak of the cycle. In my empirical analysis I will exploit variation across countries to assess the impact of a credit boom on the allocation of factors.

In recent years, a new literature focusing on factor misallocation has emerged. A crucial question is which measure should be considered to identify misallocation. Restuccia and Rogerson (2008), and Hsieh and Klenow (2009) assess the within-industry misallocation by measuring the dispersion of marginal products. Alternatively, Bartelsman, Haltiwanger and Scarpetta (2013) adopt a measure based on the covariance between size and productivity, where a weaker link denotes a worse allocation of factors. While the approach used here is similar to the latter, the analysis follows a separate line of inquiry in at least two ways. First, I rely primarily on industry-level data to detect between-industry rather than within-industry misallocation. Looking at the reallocation of factors between different industries is more appropriate to motivate my theory; it is also better suited to support the causal claims made by the empirical model set out here. Studies that measure misallocation typically deal with firm-level data and avoid between-industry considerations for comparability issues. However, the goal here is not to obtain an absolute measure of misallocation by doing an accounting of aggregate productivity, but rather to compare the allocation pathway across countries exhibiting different credit growth. The problems related to the lack of comparability of different industries are attenuated by the second point of departure from the literature: this analysis is based on growth rates rather than levels. Then, instead of looking at the correlation between size and productivity, I examine the correlation between input/output growth and productivity growth across industries and countries with a different credit growth prior to 2008.[12] Specifically, the model I will estimate is:

$$Y\_growth_{k,j} = \alpha_k\,(industry_k) + \beta_j\,(country_j)$$

$$+\delta\,(TFP\_growth_{k,j}) + \gamma\,(TFP\_growth_{k,j} \times credit\_growth_j) + controls_{k,j} + \varepsilon_{k,j}.$$

The dependent variables will include measures of growth in value added, capital, and labor for industry k in country j. $industry_k$ and $country_j$ are dummy variables respectively for industries and countries. The measure of productivity I will use is the Total Factor Productivity of each industry k in country j, $TFP\_growth_{k,j}$. Finally, $credit\_growth_j$ is the growth in aggregate credit in country j . While δ tells us about the overall correlation between productivity growth and input/output growth, γ tells us how this relation changes with credit growth. A positive γ would tell us that in those countries experiencing a larger credit boom, the effect of TFP growth on industry growth is higher. Conversely, a negative γ would work in the opposite direction: credit booms would be associated with a weaker relation between the productivity and the performance of an industry.

The measures of aggregate credit I use are from the BIS Statistics and include Credit to the Private Non-Financial Sector and Credit to Non-Financial Corporations (NFC).[13] All quantities are deflated by the CPI. To build the growth rate variables, I first took the year-by-year log-variation and multiplied by 100, and then computed the simple average from 2001 to 2007.[14] The results are reported in Figure 2.

As we can see, all countries went through a period of general credit growth with the sole exception of Germany, which reports a slight decrease in the Credit to the PNFS and to NFC during the examined period. At the opposite extreme, Ireland and Spain, notably the two countries that suffered major banking crises, experienced an outstanding credit boom, as measured by both of the two quantities.

Data on industries are derived from the EU KLEMS Growth and Productivity Accounts. The database contains industry-level measures of output, capital, employment and TFP. Measurements and computations are based on the growth accounting methodology.[15] Industrial classification is based on the NACE1, up to 32 industries. Since the focus here is on the allocation of factors to the Non-Financial Sector, I exclude from my sample the entire Finance sector. Measures of Capital and Value Added are in volume indices. The growth variables are built in the same way as those for total credit.

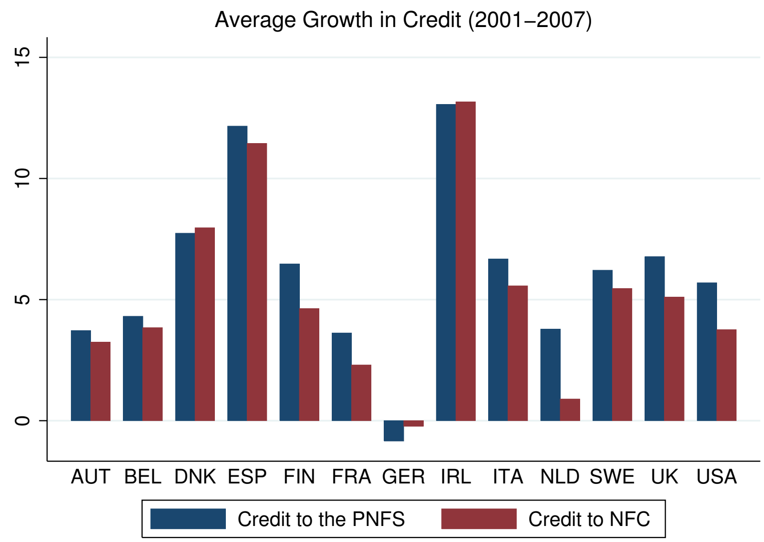
[12] Borio, Kharroubi, Upper and Zampolli (2016) provide the closest comparison to our study. The authors also look at the variation of between-industry allocation in relation to credit growth. However, they only focus on labor productivity and follow the same decomposition used by Bartelsman, Haltiwanger and Scarpetta (2013), originally introduced by Olley and Pakes (1996).

[13] In this quantity the credit to households and non-profit institutions is excluded.

[14] I chose 2001 as the starting year for my analysis since it corresponds to the bottom of business cycle for most of Western countries. However, results are robust to small changes in the starting year.

[15] See O'Mahony and Timmer (2009) for a detailed description of the dataset and the methodology.

Figure 2



Notes: Data are from the "Total credit to the non-financial sector" databases by the Bank for International Settlements.

Measures of input and output can tell us about the growth and allocation of productive factors across industries. In order to verify that the results are driven by the credit allocation channel, I integrated the data with a measure of financial leverage to be used as an additional dependent variable. Given that balance sheets data by industry are not available, I constructed a summary variable from Compustat Global and North America. For each company in the dataset I computed the average debt-to-equity ratio and its annual growth.[16] I then averaged across companies in each industry and country. Finally, I computed the average from 2001 to 2007. Note that the growth in leverage is only measured on the intensive margin without considering the entry and exit of firms in the dataset.[17]

The results for our main specification are reported in Table 1 and 2, respectively when we use the Credit to the Private Non-Financial Sector and the Credit to Non-Financial Corporations.[18] For every regression I include as a control the initial share in 2001 of the dependent variable in the total economy of the country. For the Debt-to-Equity ratio, the respective control is the initial level. I also show the results when controlling for the interaction with the initial level of credit, measured as the ratio to GDP. This is to avoid the results are driven by a convergence in levels of aggregate credit.[19] The growth in Debt-to-Equity ratio should help reveal those industries that increased their dependence on external finance. In order to avoid the variation from a change in the value of assets, I also control for the average asset growth for the respective companies

[16]The ratio is computed as (Total Liabilities)/(Total Assets-Total Liabilities). Negative values and outlying values over 25 are dropped.

[17]This is a reasonable restriction given that the Compustat database is limited to the small sub-sample of publicly traded firms.

[18]Note that the different number of observations depends on the availability of data for the different industries in the different countries. In particular, data on capital are not available for Belgium, France and Ireland.

[19]For example high-growing credit countries may have started from lower levels of credit, which may imply a negative relation between the productivity and performance of the various industries.

in the Compustat database. Finally I included the square of the TFP growth and the relative interactions in all regressions.[20]

The interaction between the TFP growth and credit growth is significantly negative in all cases except for the Debt-to-Equity ratio when I use the credit to Non-Financial Corporations and control for the initial credit to GDP level.[21] These results are in favor of the hypothesis that credit booms are associated with a worse allocation of factors between the industries. In fact, those industries which experienced a bigger increase in productivity grew relatively less in countries which experienced a more rapid aggregate credit boomed. The effect is similar when we consider the increase in financial leverage of the Compustat companies. More productive industries showed a relative increase in their Debt-to-Equity ratio when the growth in aggregate credit was lower. This suggests that a misallocation of funds could be at the origin of the misallocation of factors.

A possible critique to the results above is that they could be driven by reverse causality: those countries having a worse allocation of factors may need a bigger increase in aggregate credit to reallocate resources between the industries. In particular, credit could be optimally allocated to low productive sectors to boost long-term development and promote convergence.[22] In order to offset the likelihood of reverse causality, I proxy the TFP growth of the industries in all countries with the TFP growth of the American industries, on the assumption that the growth in productivity of the American industries can be adopted as a measure of their technological advancement. Consistent with the chosen proxy variable, it is argued that all countries should optimally invest in those sectors showing the greatest progress. The model I estimate here is similar to the previous estimation, but the productivity measure is no longer country-specific, which means that the impact of the δ is now captured by the industry-fixed effects:

$$Y\_growth_{k,j} = \alpha_k \left(industry_k\right) + \beta_j \left(country_j\right)$$

$$+\gamma \left(US\_TFP\_growth_k \times credit\_growth_j\right) + controls_{k,j} + \varepsilon_{k,j}.$$

The results are reported in Table 3 and 4, again for the Credit to the Private Non-Financial Sector and the Credit to Non-Financial Corporations. American industries are now excluded from the regressions. All the controls are similar to the previous specification. As we can see the effect of the interaction between the TFP growth in the American industries and the credit growth is always significantly negative.

The evidence set out here contradicts the proposition of the productive efficiency role of bubbly credit

[20]Results have the same (or even improved) significancy when we exclude the square terms.
[21]At the same time, the overall effect of the TFP growth is (most of the time) significantly positive.
[22]Also note that there is an alternative hypothesis that the increase in credit to an industry reduces its productivity. This would still be in favor of the misallocation result, even though at the within-industry level.

advanced by recent literature on rational bubbles. However, in the following sections, I will show that the emergence of a bubble can be a natural way to admit the misallocation of factors during a credit boom.

3 A Model of Rational Bubbles with Capital Misallocation

In this section, I will introduce the theory supporting the main claim of the paper. The central purpose is to describe the mechanism by which a rational bubble can induce a misallocation of factors and provide the necessary conditions for the misallocation result.

I will first describe the framework and characterize the equilibrium without bubbles. Then I will introduce the possibility of bubbly credit and analyze the bubbly equilibria. Note that in the setup there is no uncertainty. I will focus only on the steady state equilibria, given that the model presents trivial dynamics. I will introduce unexpected shocks and examine the dynamics for the richer model proposed in Section 4.

3.1 The Bubble-Free Environment

The model is based on the classic Over-Lapping Generations framework set out by Diamond (1965) and Tirole (1985), with two-periods (young and old) lived agents.[23] In the framework, there are three different types of agents, each of measure one:[24] Workers, High-type investors and Low-type investors. To make things more simple, I assume that all agents will only maximize their old-age consumption.

When young, workers receive a wage w.[25] While they may want to save their entire wage to consume when old, they have no technology to store it. Their only option is lending in the credit market to earn an income in the following period.

Investors, on the other hand, do not receive any wage. However, when they are born, they can install capital and rent in the following period to competitive firms owning production technologies of type H or L:

$$A_j k_{j,t} \; for \; j \in \{H, L\} \,. \tag{1}$$

Capital is specific for the two types of technologies: once installed, a given type of capital cannot be intratemporally rented to a different technology. High-type and Low-type investors differ in the type of capital they can install and, then, on the technology they can access. We assume $A_H > A_L$. We will assume that capital fully depreciates in production.

Finally, young agents in this economy can meet in a competitive credit market. Specifically, young investors can get external financing by selling credit contracts. However, in keeping with the new literature

[23]Note that qualitatively similar results could be obtained in an environment with infinitely-lived agents hit by uninsurable idiosyncratic shocks. Woodford (1990), for example, proposes an elegant way to reproduce Over-Lapping Generations behavior starting from infinitely-lived agents.

[24]Note that there is no population growth in the environment.

[25]In Section 4 workers will earn their wage by supplying labor.

on rational bubbles, a borrowing constraint limits the amount they can borrow:

$$R_{t+1}d_{j,t+1} \leq \phi MRK_{j,t+1}k_{j,t+1} \; for \; j \in \{H, L\} \tag{2}$$

with $\phi < 1$. On the left-hand side, R_{t+1} is the market interest rate, and $d_{j,t+1}$ is the debt issued by investor of type j. The promised repayment $R_{t+1}d_{j,t+1}$ cannot be higher than a fraction ϕ of the future capital income of the investor. Note that $MRK_{j,t+1}$ is the price of capital for the two types of production. This constraint is quite standard in the literature and can be interpreted as a limit on the pledgeable income of the borrower. In keeping with this literature, a binding borrowing constraint can push the interest rate below the growth rate of the economy and open the way for the existence of bubbles even if the economy is dynamically efficient.

Finally the budget constraint for an investor is:

$$k_{j,t+1} = d_{j,t+1} \; for \; j \in \{H, L\}. \tag{3}$$

We can now define the equilibrium in this economy.

DEFINITION: *A competitive equilibrium is a list of consumption, debt, capital, labor, and prices such that:*

(i) Young workers maximize their old-age consumption by buying credit contracts in the value of w. Old workers consume $R_t w$

(ii) Young investors choose $k_{j,t+1}$ and $d_{j,t+1}$, given prices $(R_{t+1}, MRK_{j,t+1})$, maximizing future profits

$$MRK_{j,t+1}k_{j,t+1} - R_{t+1}d_{j,t+1} \; for \; j \in \{H, L\} \tag{4}$$

subject to budget constraints (3), borrowing constraints(2) and resource constraints

$$d_{j,t+1} \geq 0.$$

Old investors consume their profits

(iii) Factors are paid at their marginal productivity:

$$MRK_{j,t} = A_j \; for \; j \in \{H, L\}. \tag{5}$$

(iv) All markets clear in every period. In particular, it must be:

$$d_{H,t+1} + d_{L,t+1} = w. \tag{6}$$

In this stylized economy with linear production technologies and borrowing constraints, High-type and Low-type investors can respectively offer rates ϕA_H and ϕA_L. In equilibrium, it must be $R^* = \phi A_H$ with only High-type investors obtaining funds in the credit market. Then, all capital is optimally allocated to the High-type production: $d_H^* = k_H^* = w$. Aggregate production and consumption are $Y^* = A_H w$ and $C = R^* w + (1 - \phi) Y^* = Y^* = A_H w$.

In the next section, I will analyze how the emergence of a bubble distorts the allocation of capital in this economy. In order to introduce bubbles I will make the following assumption:

ASSUMPTION 1: $\phi < \frac{1}{A_H} \rightarrow R^* < 1$.

This is the traditional condition for the existence of bubbles: the interest rate must be lower than the growth rate of the economy. It is clear that R^* can be lower than 1, even if the economy is dynamically efficient, i.e., $A_H > 1$. In the following section, I will show how the effect of a bubble on the allocation of capital depends on the market interest rate and the returns on capital A_H and A_L.

3.2 Introducing Bubbly Debt

A bubble is an asset with no fundamental value, i.e., essentially a pyramid scheme. A young agent would buy a bubbly asset only with the purpose of reselling it in the following period. Usually, according to the literature on rational bubbles, the stock of bubbly assets is given and the analysis is focused on the exchange. Martin and Ventura (2012) introduced the possibility of issuing new bubbly assets or starting a new pyramid scheme. This aspect is relevant because the agent that introduces a new bubbly asset in the economy earns a windfall. As we will see, the privilege of being an issuer of bubbly assets is crucial for our misallocation result.

I will assume bubbles can be issued only by young investors.[26] A bubble can be interpreted as a credit note, apparently identical to the other credit notes secured by the future pledgeable income of the investors. The main difference is that the bubbly notes will not be repaid by borrowers, but will instead be repaid with the purchase of the future generation of workers.

[26]The assumption does not affect the qualitative results of my analysis. In the next section I will introduce a rationale for investors being the only possible issuers of bubbly debt.

Credit markets are still competitive. However, now an investor can issue two types of debt, secured and unsecured. For $j \in \{H, L\}$, these debt types are defined as:

$$d^S_{j,t+1} = \begin{cases} d_{j,t+1} & if \quad R_{t+1} d_{j,t+1} \leq \phi MRK_{j,t+1} k_{j,t+1} \\ \frac{\phi}{R_{t+1}} MRK_{j,t+1} k_{j,t+1} & if \quad R_{t+1} d_{j,t+1} > \phi MRK_{j,t+1} k_{j,t+1} \end{cases} \tag{7}$$

$$d^U_{j,t+1} = \begin{cases} 0 & if \quad R_{t+1} d_{j,t+1} \leq \phi MRK_{j,t+1} k_{j,t+1} \\ d_{j,t+1} - \frac{\phi}{R_{t+1}} MRK_{j,t+1} k_{j,t+1} & if \quad R_{t+1} d_{j,t+1} > \phi MRK_{j,t+1} k_{j,t+1}. \end{cases} \tag{8}$$

With the choice of secured funding, the investor will now face the following borrowing constraint:

$$R_{t+1} d^S_{j,t+1} \leq \phi MRK_{j,t+1} k_{j,t+1} \; for \; j \in \{H, L\}. \tag{9}$$

It is worthy to stress that both secured and unsecured notes must promise the same return R_{t+1} to be purchased in equilibrium.

When an investor can issue unsecured debt he earns a windfall, since he will not be responsible for its repayment. The budget constraint of an investor can now be rewritten as:

$$k_{j,t+1} + l^U_{j,t+1} = d^S_{j,t+1} + d^U_{j,t+1} \; for \; j \in \{H, L\} \tag{10}$$

where $l^U_{j,t+1}$ represents the purchase of unsecured notes by investors of type j. It is relevant to observe that the possibility of issuing unsecured debt depends on the beliefs of the agents in the economy. In our framework an investor cannot actively influence these beliefs. This means that he can choose $d^S_{j,t+1}$ but not $d^U_{j,t+1}$.

A bubbly scheme is sustainable if the future generations of agents have enough income to repurchase the unsecured notes issued in the market. The equilibrium interest rate is linked to the path of the unsecured debt by the following market clearing relation:

$$R_{t+1} \left(l^U_{H,t+1} + l^U_{L,t+1} + w_t - d^S_{H,t+1} - d^S_{L,t+1} \right) = w_{t+1} - \left(k_{H,t+2} + k_{L,t+2} \right). \tag{11}$$

The left-hand side represents the $t+1$-value of all unsecured notes issued before time $t+1$; the right-hand side represents the available income at time $t+1$ that young agents do not invest in capital.

We can define the competitive equilibrium when there is bubbly debt in the economy.

DEFINITION: *A competitive equilibrium with bubbly debt is a list of consumption, secured and unse-*

cured debt, capital, labor, and prices such that:

(i) Young workers maximize their old-age consumption by buying credit contracts in the value of w*. Old workers consume* $R_t w$*.*

(ii) Young investors choose $k_{j,t+1}$*,* $d^S_{j,t+1}$ *and* $l^U_{j,t+1}$*, given* $d^U_{j,t+1}$ *and prices* $(R_{t+1}, MRK_{j,t+1})$*, maximizing future profits*

$$MRK_{j,t+1}k_{j,t+1} - R_{t+1}\left(d^S_{j,t+1} - l^U_{j,t+1}\right) \; for \; j \in \{H, L\} \tag{12}$$

subject to budget constraints (10)*, borrowing constraints* (9) *and resource constraints*

$$d^S_{j,t+1} \geq -d^U_{j,t+1} \; and \; l^U_{j,t+1} \geq 0.$$

Old investors consume their profits

(iii) Factors are paid at their marginal productivity:

$$MRK_{j,t} = A_j \; for \; j \in \{H, L\}\,. \tag{13}$$

(iv) Agents hold beliefs about the path of $d^U_{H,t+1}$ *and* $d^U_{L,t+1}$

(v) All markets clear in every period. In particular, it must be:

$$R_t\left(l^U_{H,t} + l^U_{L,t} + w_{t-1} - d^S_{H,t} - d^S_{L,t}\right) = w_t - \left(k_{H,t+1} + k_{L,t+1}\right). \tag{14}$$

In this section, I will characterize the steady state equilibria with bubbly debt. An equilibrium with bubbly debt is supported by the beliefs of the agents, which in turn determine the equilibrium rate R^b. Furthermore, these beliefs also determine who can issue bubbly debt. This aspect is critical to understanding our misallocation result. The investors issuing unsecured debt earn a rent which they can use to increase their investment. Particularly, this issuing ability has nothing to do with the actual productivity of the issuer. In what follows I will assume that H-type and L-type investors always issue a fraction $(1-\delta)$ and δ of the total value of new unsecured notes $d^U_H + d^U_L$.

In steady state, bubbly debt can exist only if the equilibrium rate R^b is higher than $R^* = \phi A_H$. In fact, if $R^b = R^*$, we know from the previous section that it must be $w = d^S_H$, i.e., the H-type investors would be able to secure the entire lending amount from the workers.

Investors choose to be borrowers or lenders in the credit market depending on whether their return A_j is higher or lower than the market interest rate. I will describe the equilibria with bubbly debt in the following three cases: $R^* < R^b \leq A_L$, $A_L < R^b \leq A_H$ and $A_H < R^b$.[27]

CASE 1*:* $R^* < R^b \leq A_L$

If R^b is lower than A_L, both H-type and L-type investors want to be net borrowers in the credit market. This also implies $l_H^U = l_L^U = 0$, i.e., investors do not want to hold bubbly notes. The quantity $w - d_H^S + d_L^S$ then represents the aggregate value of bubbly debt in the economy, which is the value of newly and previously issued unsecured notes. Specifically, this quantity cannot be entirely transferred to the investors in the form of new unsecured debt - a part of it must be used to repurchase the existing unsecured debt. From market clearing condition (14) we can solve for the total steady state value of new unsecured debt issued by the investors:

$$d_H^U + d_L^U = \left(1 - R^b\right)\left(w - d_H^S - d_L^S\right). \tag{15}$$

In this last equation, we find the traditional necessary condition for the sustainability of a bubbly equilibrium: $R^b \leq g = 1$. A higher R^b reduces the amount of new unsecured debt the investors can issue since the workers need to use a larger share of their income to buy existing credit notes. This is in keeping with the characteristic crowding-out effect of rational bubbles. In the extreme case of $R^b = 1$, there is no unsecured transfer from the workers to the investors in the steady state.

The equilibrium H-type and L-type capital are given by:

$$k_H = d_H^S + d_H^U = \frac{\phi}{R^b} A_H k_H + (1-\delta)\left(1 - R^b\right)\left[w - \frac{\phi}{R^b}\left(A_H k_H + A_L k_L\right)\right] \tag{16}$$

$$k_L = d_L^S + d_L^U = \frac{\phi}{R^b} A_L k_L + \delta\left(1 - R^b\right)\left[w - \frac{\phi}{R^b}\left(A_H k_H + A_L k_L\right)\right]. \tag{17}$$

With respect to the equilibrium without bubbles, now the the Low-type investors can raise financing in the credit market as long as $\delta > 0$ and $R^b < 1$. Moreover, the Low-type investors will invest their rent in L-type capital given that the market rate R^b is lower than A_L. This, eventually, raises also the amount of secured debt issued by the Low-type investors. In the case of $R^b > A_L$, a Low-type investor who issues unsecured notes would use his rent to purchase credit contracts in the market instead.

We can solve further for k_H and $k_L$to obtain:

$$k_H = \frac{(1-\delta)\,\frac{1-R^b}{R^b}\,\frac{R^b - \phi A_L}{1-\phi A_L}}{1 - \frac{\phi A_H}{R^b} + (1-\delta)\,\frac{1-R^b}{R^b}\,\frac{\phi A_H - \phi A_L}{1-\phi A_L}} w \tag{18}$$

$$k_L = \frac{\delta\,\frac{1-R^b}{R^b}\,\frac{R^b - \phi A_H}{1-\phi A_H}}{1 - \frac{\phi A_L}{R^b} + \delta\,\frac{1-R^b}{R^b}\,\frac{\phi A_L - \phi A_H}{1-\phi A_H}} w. \tag{19}$$

[27]Note that the existence of the three intervals of equilibria is subject to $R^b \leq 1$.

In the bubble-free environment, the higher productivity was driving the allocation of financing and the installment of capital in the High-type sector. Here, secured debt still depends on A_H and A_L. However, the relative allocation of unsecured funding has no relation to the productivity of the borrower - it is completely driven by the agents' beliefs about δ. In particular, an increase in δ expands the relative allocation of capital in favor of the Low-type sector.

CASE 2: $A_L < R^b \leq A_H$

When R^b is higher than A_L but lower than A_H, only the High-type investors will be net borrowers in the credit market. Young Low-type investors will sell their unsecured notes to purchase credit contracts in the market, or they will simply keep their unsecured notes and sell them when old. In steady state, the market clearing condition (14) becomes:

$$d_H^U + R^b d_L^U = \left(1 - R^b\right)\left(w - d_H^S\right). \tag{20}$$

Given $d_L^U = \frac{\delta}{1-\delta} d_H^U$, the value of new unsecured debt issued by the High-type investors is:

$$d_H^U = \frac{1-\delta}{1-\delta\left(1-R^b\right)}\left(1-R^b\right)\left(w - \frac{\phi}{R^b} A_H k_H\right). \tag{21}$$

Aggregate High-type capital is:

$$k_H = d_H^S + d_H^U = \frac{1-\delta\left(1-R^b\right)-R^b}{1-\delta\left(1-R^b\right)-\phi A_H} w. \tag{22}$$

In this second case, there is no capital accumulated in the Low-type sector. A higher δ reduces the amount of High-type capital as it increases the rent consumed by the Low-type investors.

CASE 3: $A_H < R^b$

When the interest rate is higher than A_H, both types of young investors want to be net lenders in the market. In this scenario it must be $d_H^S = d_L^S = k_H = k_L = 0$. From the market clearing condition (14), all resources are employed to purchase existing secured notes in every time:

$$R^b\left(d_H^U + d_L^U\right) = \left(1 - R^b\right) w. \tag{23}$$

We can now summarize our results:

$$k_H = \begin{cases} w & if\ R^b = R^* \\ \frac{(1-\delta)\frac{1-R^b}{R^b}\frac{R^b-\phi A_L}{1-\phi A_L}}{1-\frac{\phi}{R^b}A_H+(1-\delta)\frac{1-R^b}{R^b}\frac{\phi A_H-\phi A_L}{1-\phi A_L}}w & if\ R^* < R^b \leq A_L \\ \frac{1-\delta\left(1-R^b\right)-R^b}{1-\delta(1-R^b)-\phi A_H}w & if\ A_L < R^b \leq A_H \\ 0 & if\ R^b > A_H \end{cases}, \tag{24}$$

$$k_L = \begin{cases} 0 & if\ R^b = R^* \\ \frac{\delta\frac{1-R^b}{R^b}\frac{R^b-\phi A_H}{1-\phi H}}{1-\frac{\phi}{R^b}A_L+(1-\delta)\frac{1-R^b}{R^b}\frac{\phi A_L-\phi A_H}{1-\phi A_H}}w & if\ R^* < R^b \leq A_L \\ 0 & if\ A_L < R^b \leq A_H \\ 0 & if\ R^b > A_H \end{cases}. \tag{25}$$

Figures 3, 4 and 5 plot the allocation of High-type and Low-type capital against R^b if $1 \leq A_L < A_H$, $A_L < 1 \leq A_H$ and $A_L < A_H < 1$, given $\delta = 0.5$. The figures confirm that the emergence of a bubble misallocates capital only if $R^b < A_L$. We can state the following Proposition.

PROPOSITION 1: *A necessary condition for bubbles inducing a misallocation of factors is* $R^b < A_L$.

We also want to examine how the bubble affects the aggregate accumulation of capital, the output and the welfare of the economy.

PROPOSITION 2: *The emergence of a bubble always reduces aggregate output and capital. The effect on aggregate consumption can be positive only if* $A_H < 1$.

The proof of Proposition 2 is in Appendix 1. Figures 6, 7 and 8 plot the steady state values of total output and consumption against R^b, given $\delta = 0.5$. In this model, a bubble is always contractionary. This result does not only derive from the typical crowding-out effect. The model adds an additional contractionary effect associated with the misallocation of factors. However, a bubble may still increase aggregate consumption, but only when the economy is dynamically inefficient.[28]

Finally, we can analyze how the identity of the investor that issues bubbly debt influences the aggregate economy.

PROPOSITION 3: *An increase in* δ *always reduces aggregate output and capital. The effect on aggregate consumption can be positive only if* $A_H < 1$.

[28] This is in line with the original theory by Tirole.

Proposition 3 is proved in Appendix 2. Intuitively, when Low productivity investors issue a larger share of unsecured notes, factors are misallocated and output is lower. In addition, since Low-type investors earn lower returns, a larger share of the workers' future endowment must be allocated to the repayment of bubbly debt. This, eventually, reduces the total stock of capital.

To conclude, the model described here adds a new dimension to existing theories of rational bubbles. Bubbles do not only affect the aggregate accumulation of capital, they also have a re-allocation effect. Productive factors can be crowded out from specific sectors to be re-allocated to others. For a given interest rate, the effect of a bubble depends on this re-allocation of factors. In particular, the cost of a bubbly episode may be higher if it involves a large misallocation of capital towards low productive sectors.

This model, however, still does not tell us which investors would have an advantage in the issuing of bubbly debt. In addition, the contraction in output is always associated with a reduction in the stock of capital. In the next section, I will introduce some risk in the activity of the investors - which will affect their life expectancy on the market and, thereafter, their ability to maintain a bubbly scheme and accumulate capital over time. I will show that a bubble can boost aggregate capital accumulation even if that induces a misallocation of factors and a decrease in total production.

4 Credit Bubbles and Misallocation in a Model with Risky Investments

In this section I will extend the previous model by introducing a mechanism which predicts the misallocation equilibrium in a unique way. Importantly, the same mechanism will also open the doors for capital accumulation even if the bubble is contractionary. Here, investors will live for more than two periods but they will face some risk in their investment activity which will affect their life expectancy on the market and, thereafter, their ability to maintain a bubbly scheme and accumulate capital over time. In addition, workers will now supply labor and make an intertemporal consumption choice when young.[29] All agents are assumed to be risk neutral.

I will describe the problem faced by workers and investors in the following subsections. Note that, for simplicity, agents behave as if bubbles were deterministic. In looking at the dynamics, I will assume that the shocks to the system are unexpected.

[29]This aspect will be relevant when introducing nominal rigidities.

4.1 OLG Workers

Workers live for two periods as in the previous version of the model. However, they now choose their total labor supply and consumption in both their young and old periods, by maximizing the following utility:

$$\log\left(c_{Y,t}\right) - \varphi h_t + \log\left(c_{O,t+1}\right) \tag{26}$$

subject to $c_{Y,t} = w_t h_t - l_{t+1}$ and $c_{O,t+1} = R_{t+1} l_{t+1}$, where l_{t+1} denotes lending in the credit market. For simplicity I assume that the disutility from working is linear. The solution to the problem gives the aggregate supply of labor and lending:

$$h_t = \frac{2}{\varphi} \tag{27}$$

$$l_{t+1} = \frac{1}{2} w_t h_t = \frac{w_t}{\varphi}. \tag{28}$$

4.2 Risky Investments

Investors are still grouped into two categories of mass one - High-type and Low-type - but they now live for more than two periods. Specifically, each investor has an i.i.d. probability σ of surviving in each period t. Then, in each period a mass $(1-\sigma)$ of old investors leave the market and the same number of new investors enter the market with endowment e.[30] Similarly to the previous section, the investors want to maximize their consumption in their last period of life.[31] Then, in all the previous periods, they will always reinvest and continue to accumulate capital.

Again, the investors have a storing technology that will allow the installation of a specific kind of capital to rent in the following period to High-type or Low-type production. Unlike the activities described in the previous section, here the storing activity is risky. In particular, with respective probabilities $(1-\varepsilon_H)$ and $(1-\varepsilon_L)$, the storing can fail and the investor can end up with no capital in the following period. I make the assumption that these shocks are idiosyncratic and not insurable.

Production functions are now Cobb-Douglas combining capital and labor:

$$A_j k_{j,t}^{\alpha} h_{j,t}^{1-\alpha} \; for \; j \in \{H, L\}. \tag{29}$$

I make the following assumptions:

[30] Borrowing banks are modeled in a similar fashion in the model of bank runs described by Gertler and Kiyotaki (2015).

[31] Note that the same decision would derive if investors maximized a linear utility over consumption in different periods, $\sum_{t=0}^{\infty} c_{m,t}$, and the return from borrowing and investing was always higher than 1.

ASSUMPTION 2: $\varepsilon_H < \varepsilon_L$.

ASSUMPTION 3: $\varepsilon_H^\alpha A_H > \varepsilon_L^\alpha A_L$.

Assumption 2 states that the probability of failing is higher for an H-type investor. Nonetheless, Assumption 3 confirms that the overall H-type productivity is still higher. These premises describe an environment in which higher productivity sectors are also riskier. Conversely, low productive sectors offer more stability over time. Then, the two types of investment offer a different combination in the risk-return spectrum.

An investor m, of type H or L, raises external funding in the credit market and faces a similar borrowing constraint:

$$R_{t+1} d_{m,t+1}^S \leq \phi MRK_{j,t+1} \varepsilon_j i_{m,t+1} \; for \; j \in \{H, L\}, \tag{30}$$

where $d_{m,t+1}^S$ and $i_{m,t+1}$ are secured debt and investment. That is to say, an investor of type j can secure his borrowing up to a fraction ϕ of his expected capital income. The investors can also expand their borrowing by issuing bubbly debt. At this point, a further restriction is imposed:

ASSUMPTION 4: *A debt contract can be exchanged as long as the issuer has positive equity.*

Assumption 4 comes with an important implication: when an investor fails or dies, all the bubbly notes that he has issued will burst.[32] This is a more accurate description of what happens in the real world where tradable securities fail automatically with their issuers' failure, or where financial institutions issue short-term notes which are rolled over under the same roof. Assumption 4 introduces a gap in the expected duration of H-type and L-type activities. It is worth pointing out that, given that H-type investors experience a shorter life expectancy on the market, they have a lower probability of rolling over a bubbly scheme.

4.3 Equilibrium and Steady State Solutions

The equilibrium in the economy is now defined as follows:

DEFINITION: *A competitive equilibrium is a list of consumption, lending, secured and unsecured debt, capital, labor, and prices such that:*

(i) Young workers maximize their utility (26) *by choosing* h_t *and* l_{t+1}. *Old workers consume* $R_t l_t$

(ii) An investor m *of type* j *who is still active in the market in period* t *chooses* $i_{m,t+1}$, $l_{m,t+1}^U$ *and* $d_{m,t+1}^S$, *given* $d_{m,t+1}^U$ *and prices* $(R_{t+1}, MRK_{j,t+1})$, *maximizing profits in the last period of his life*

$$\sum_{q=1}^{\infty} (1-\sigma)\, \sigma^{q-1} \varepsilon_j \left[MRK_{j,t+q} i_{m,t+q} - R_{t+q} \left(d_{m,t+q}^S - l_{m,t+1}^U \right) \right] \; for \; j \in \{H, L\} \tag{31}$$

[32] Note that an investor whose storage activity fails will also end up with zero consumption.

subject to budget constraint

$$c_{m,t} + i_{m,t+1} + l^U_{m,t+1} = MRK_{j,t} i_{m,t} - R_t d^S_{m,t} + d^S_{m,t+1} + d^U_{m,t+1}, \tag{32}$$

borrowing constraint (30) *and resource constraints*

$$d^S_{m,t+1} \geq -\left(MRK_{j,t} i_{m,t} - R_t d^S_{m,t} + d^U_{m,t+1}\right) \text{ and } l^U_{m,t+1} \geq 0.$$

An investor who dies in period t*, consumes his final income* $c_{m,t} = MRK_{j,t} i_{m,t} - R_t \left(d^S_{m,t} - l^U_{m,t}\right)$*, while an investor who fails leaves the market with no final consumption*

(iii) Factors are paid at their marginal productivity:

$$w_t = (1-\alpha) A_H \left(\frac{k_{H,t}}{h_{H,t}}\right)^{\alpha} = (1-\alpha) A_L \left(\frac{k_{L,t}}{h_{L,t}}\right)^{\alpha} \tag{33}$$

$$MRK_{j,t} = \alpha A_j \left(\frac{h_{j,t}}{k_{j,t}}\right)^{1-\alpha} \; for \; j \in \{H, L\} \tag{34}$$

with $k_{j,t} = \varepsilon_j \int_{m \in j} i_{m,t} \; for \; j \in \{H, L\}$

(iv) Agents hold beliefs about the path of $d^U_{j,t+1}$ $for \; j \in \{H, L\}$

(v) All markets clear in every period.

As described in the previous section, the necessary condition to have bubbles misallocating resources is that both borrowing constraints are binding. Therefore, I make the following assumption.

ASSUMPTION 5: $R_{t+1} < \varepsilon_L MRK_{L,t+1} \; \forall \; t.$ [33]

Note that Assumption 5 implies $R_{t+1} < \varepsilon_H MRK_{H,t+1}$ a fortiori. All investors will try to borrow until their constraints bind and only the workers will lend in the credit market.

I begin by characterizing the steady state equilibria of the economy. Without bubbles in the economy, the steady state interest rate is:[34]

$$R^* = 2\phi \frac{\alpha}{1-\alpha}. \tag{36}$$

The previous section described bubbly debt equilibria as possible if R^b was lower than the growth rate of the economy. This was possible because a debt security could also be exchanged after the death of the issuer. Here a bubbly scheme will burst if the issuer dies or fails, which means that in a steady state with

[33] The condition is on variables endogenously determined in the model. Therefore, it implicitly sets restrictions on parameters so that all the equilibria we will characterize (with or without bubbles) respect the inequality.

[34] We can solve by plugging (33) in

$$R^* \frac{w}{\varphi} = \phi\alpha \left(A_H k^{\alpha}_H h^{1-\alpha}_H + A_L k^{\alpha}_L h^{1-\alpha}_L\right). \tag{35}$$

bubbles, High-type and Low-type investors cannot promise a return higher than $\sigma\varepsilon_H$ and $\sigma\varepsilon_L$.[35] From now on, I will make the following assumption:

ASSUMPTION 6: $\sigma\varepsilon_H \leq R^* < \sigma\varepsilon_L$.

Assumption 6 implies that only L-type investors can run a bubbly scheme in steady state. By backward induction, only L-type investors can credibly initiate a bubbly scheme because their survival rate in the market is higher given a lower probability of failure. A rational bubbly scheme relies on the expectation that the agents will continue to buy in the long run. Borrowers with riskier projects have a lower probability of survival and cannot sustain a long term pyramid scheme. In this course of event, a bubble will necessarily prompt the misallocation of resources from higher to lower productive borrowers. The interest rate R^b in this bubbly equilibria will be such that $R^* \leq R^b \leq \sigma\varepsilon_L < 1$.

The dynamics of aggregate capital can now be set out in both sectors:

$$k_{H,t+1} = \varepsilon_H \left\{ (1-\sigma) e + \sigma (1-\phi) \frac{\alpha}{1-\alpha} w_t h_{H,t} + \frac{\phi}{R_{t+1}} \frac{\alpha}{1-\alpha} w_{t+1} h_{H,t+1} \right\} \tag{37}$$

$$\begin{aligned} k_{L,t+1} = {} & \varepsilon_L \left\{ (1-\sigma) e + \sigma (1-\phi) \frac{\alpha}{1-\alpha} w_t h_{L,t} + \frac{\phi}{R_{t+1}} \frac{\alpha}{1-\alpha} w_{t+1} h_{L,t+1} \right\} \\ & + \varepsilon_L \left\{ l_{t+1} - \frac{\phi}{R_{t+1}} \frac{\alpha}{1-\alpha} w_{t+1} h_{t+1} - R_t \left(l_t - \frac{\phi}{R_t} \frac{\alpha}{1-\alpha} w_t h_t \right) \right\}. \end{aligned} \tag{38}$$

The curly braces refer to the H-type and L-type aggregate investments in time t. Newly-arrived investors of both types invest their endowment e. Pre-existing investors who remain in the market in period t, on aggregate reinvest their income: $(1-\phi) MRK_{j,t} k_{j,t} = (1-\phi) \frac{\alpha}{1-\alpha} w_t h_{j,t}$ for $j \in \{H, L\}$. All investors in the market will also invest all external funding they are able to raise in the credit market: $\frac{\phi}{R_{t+1}} MRK_{j,t+1} k_{j,t+1} = \frac{\phi}{R_{t+1}} \frac{\alpha}{1-\alpha} w_{t+1} h_{j,t+1}$ for $j \in \{H, L\}$. In addition, L-type investors can invest the rent they obtain from issuing unsecured debts. In the last line we can see that the rent is given by the portion of current unsecured debt that is not allocated to the repayment of past unsecured debt. In an equilibrium with no bubbles, the rent is equal to 0. Finally, both types of aggregate investments are fractioned by the respective storage survival rate.

In steady state the two equations can be simplified:

$$k_H = \varepsilon_H \left\{ (1-\sigma) e + \left[\sigma (1-\phi) + \frac{\phi}{R^b} \right] \frac{\alpha}{1-\alpha} w h_H \right\} \tag{39}$$

$$k_L = \varepsilon_L \left\{ (1-\sigma) e + \left[\sigma (1-\phi) + \frac{\phi}{R^b} \right] \frac{\alpha}{1-\alpha} w h_L + \left(1 - R^b\right) \frac{R^b - R^*}{R^b} \frac{w}{\varphi} \right\}. \tag{40}$$

[35] Note that the implicit assumption is that unsecured funds are randomly allocated inside the mass of H-type and L-type investors who are in the market at time t.

Substituting into (33), the steady state labor allocation is finally obtained as a function of w :

$$h_H = \frac{(1-\sigma)\, e}{\left(\frac{w}{(1-\alpha)\varepsilon_H^\alpha A_H}\right)^{\frac{1}{\alpha}} - \left[\sigma\,(1-\phi) + \frac{\phi}{R^b}\right] \frac{\alpha}{1-\alpha} w} \tag{41}$$

$$h_L = \frac{(1-\sigma)\, e + \left(1-R^b\right) \frac{R^b - R^*}{R^b} \frac{w}{\varphi}}{\left(\frac{w}{(1-\alpha)\varepsilon_L^\alpha A_L}\right)^{\frac{1}{\alpha}} - \left[\sigma\,(1-\phi) + \frac{\phi}{R^b}\right] \frac{\alpha}{1-\alpha} w}. \tag{42}$$

It is easy to see that in the bubble-free equilibrium, i.e. when $R^b = R^*$, the allocation of capital and labor is driven by the aggregate productivities $\varepsilon_H^\alpha A_H$ and $\varepsilon_L^\alpha A_L$. Since the latter is smaller, High-type investors receive more capital and labor. The rise of a bubble misallocates factors in favor of the Low-type investors.

The following Proposition can now be stated.

PROPOSITION 4: *A bubble always reduces total output.*

A formal proof is provided in the Appendix. Intuitively, it would seem that a bigger R^b increases the amount of unsecured debt in the economy, which would raise both the misallocation and the crowding-out of capital. As expected, bubbles in this section are always contractionary. Nevertheless, this does not necessarily imply a reduction in the aggregate stock of capital as it did in the previous section.

PROPOSITION 5: *There exist steady state equilibria with bubbles in which aggregate capital increases.*

The proposition is proved in Appendix 4. The bubbly episodes preceding a financial crisis are typically characterized by a fast accumulation in capital. In particular, in the years prior to 2008 we saw a boom in the housing sector. The original theory of rational bubbles could not explain this phenomenon. In Tirole's framework, a bubble would reduce capital when the economy is dynamically inefficient. The addition of credit constraints in the new literature on rational bubbles, has introduced a new class of bubbly equilibria: by improving the intratemporal allocation of funding, bubbles can boost output and capital. In this section I introduced a further type of bubbly episode. This bubble reduces output and increases capital by misallocating resources towards low productive sectors which, nonetheless, have a higher propensity to accumulate. Our result is driven by the assumption that low productive sectors have a lower fundamental risk. Interestingly, the model predicts the emergence of non-fundamental risk in sectors that are fundamentally more stable.

4.4 The Dynamics of the Model without Nominal Rigidities

This section will set out the simulated dynamics of my model when the system is hit by unexpected shocks to the interest rate R_t. Specifically, I will analyze the transition dynamics between the bubble-free steady state, characterized by R^*, and the bubbly steady state with $R^b = \sigma\varepsilon_L$.

The model is solved numerically. The selection of parameters respects the assumptions set out in the previous section. In order to confirm Proposition 5, ε_H is set sufficiently small relatively to ε_L that a reallocation of funding towards L-type investors would boost capital accumulation.[36] In this simulation the economy starts from a bubble-free steady state: in period 11 the interest rate rises from R^* to $R^b = \sigma\varepsilon_L$; in period 71 it drops from $R^b = \sigma\varepsilon_L$ to R^*.

Figure 9 and 10 report the path for the allocation of capital and labor. While the reallocation in the labor market is symmetrical, given a fixed total labor supply, we can see how the increase in the amount of L-type capital overtakes the reduction in H-type capital when the bubble appears. This can also be observed in the path for aggregate capital presented in Figure 11. However, the rise in the aggregate stock of capital is not reflected in a long run expansion in output. Total production gradually decreases at the emergence of the bubble, and only returns to its initial steady state level when the bubble bursts (Figure 12).

An apparent drawback to this model is the timing of the expansion and the recession. Bubbly times are generally expansionary, at least in the short run; recessions typically start at the burst of the bubble. In the next section I will add nominal rigidities to our environment and show how a rise and drop in nominal returns can induce both a short-run demand effect and a long-run reallocation of factors.

4.5 The Dynamics of the Model with Nominal Rigidities

To introduce demand effects associated with nominal rigidities, I will now assume that firms produce differentiated goods and compete in a monopolistic fashion. Specifically, agents in the economy consume two identical composite goods produced by firms of type H and L:

$$Y_t = Y_{H,t} + Y_{L,t} = \left[\int_{n\in H} y_{n,t}^{\frac{\eta-1}{\eta}}\, dn\right]^{\frac{\eta}{\eta-1}} + \left[\int_{n\in L} y^{\frac{\eta-1}{\eta}}\, dn\right]^{\frac{\eta}{\eta-1}} \tag{43}$$

for $\eta \geq 1$, where $y_{n,t}$ is the output of a single firm, while $Y_{H,t}$ and $Y_{L,t}$ are the aggregate output from High-type and Low-type firms. The implied composite prices are:

$$P_t = P_{H,t} = P_{L,t} = \left[\int_{n\in H} p_{n,t}^{1-\eta} dn\right]^{\frac{1}{1-\eta}} = \left[\int_{n\in L} p_{n,t}^{1-\eta} dn\right]^{\frac{1}{1-\eta}}. \tag{44}$$

In what follows, all variables in nominal terms will have a superscript N. In order to keep the study of the dynamics as simple as possible I will keep the assumption that shocks to the system are unexpected.

I assume young workers own the monopolistic firms and earn their profits. Then, they maximize:

[36] Specifically: $A_H = 1.8$, $A_L = 0.6$, $\varepsilon_H = 0.2$, $\varepsilon_L = 1$, $\sigma = 0.5$, $\alpha = 0.65$, $\phi = 0.125$, $e = 0.001$.

$$\log\left(c_{Y,t}\right) - \varphi h_t + \log\left(c_{O,t+1}\right) \tag{45}$$

subject to $c_{Y,t} = \frac{w_t^N}{P_t} h_t + \frac{\Pi_t^N}{P_t} - l_{t+1}$ and $c_{O,t+1} = \left(R_{t+1}^N \frac{P_t}{P_{t+1}}\right) l_{t+1}$, where Π_t^N and R_{t+1}^N respectively denotes the nominal profits and interest rate. Young workers choose the following optimal supply of labor and lending:

$$h_t = \frac{w_t^N h_t}{\Pi_t^N + w_t^N h_t} \frac{2}{\varphi} \tag{46}$$

$$l_{t+1} = \frac{1}{2}\left(\frac{w_t^N}{P_t} h + \frac{\Pi_t^N}{P_t}\right) = \frac{w_t}{\varphi}. \tag{47}$$

It is worthy to note that the labor supply increases with the relative share of labor income to profits. This is crucial to generating demand effects in the economy.

Each firm n maximizes its profits

$$P_{n,t} A_j k_{n,t}^{\alpha} h_{n,t}^{1-\alpha} - MRK_{j,t}^N k_{n,t} - w_t^N h_{n,t} \; for \; j \in \{H, L\} \tag{48}$$

given prices $MRK_{j,t}^N$ and w_t^N, and demand constraint $y_{n,t} = \left(\frac{P_{n,t}}{P_t}\right)^{-\eta} Y_{j,t}$. I assume that the price of a good is set one period in advance: as long as no unexpected shock hits the economy, a firm will set the price at a constant markup $\frac{\eta}{\eta-1}$ over its marginal cost. Optimal capital and labor demand will be such that

$$\lambda_{n,t} P_{n,t} \alpha A_j \left(\frac{h_{n,t}}{k_{n,t}}\right)^{1-\alpha} = MRK_{j,t}^N \; for \; j \in \{H, L\} \tag{49}$$

$$\lambda_{n,t} P_{n,t} \left(1-\alpha\right) A_j \left(\frac{k_{n,t}}{h_{n,t}}\right)^{\alpha} = w_t^N \; for \; j \in \{H, L\}, \tag{50}$$

where $\lambda_{n,t}$ is the portion of revenues allocated to the payment of factors. Note that, when a firm can optimally set his price, it must be $\lambda_{n,t} = \frac{\eta-1}{\eta} \; \forall \; n$[37]. The aggregate λ_t in period t can be defined as

$$\lambda_t = \lambda_{H,t} \frac{Y_{H,t}}{Y_t} + \lambda_{L,t} \frac{Y_{L,t}}{Y_t}, \tag{51}$$

where $\lambda_{H,t}$ and $\lambda_{L,t}$ are the respective shares of High and Low type firms. Fluctuations in λ_t will be associated to demand effects. The incomes and profits in the economy can be rewritten as a function of λ_t and aggregate output Y_t^N: $MRK_{H,t}^N k_{H,t} + MRK_{L,t}^N k_{L,t} = \lambda_t \alpha Y_t^N$, $w_t^N h_t = \lambda_t \left(1-\alpha\right) Y_t^N$ and $\Pi_t^N = \left(1-\lambda_t\right) Y_t^N$. Then,

[37] $\lambda_{n,t}$ is indeed the inverse of the firm's markup.

from (46), I can express the labor supply as an increasing function of λ_t:

$$h_t = \frac{(1-\alpha)\lambda_t}{(1-\lambda_t)+(1-\alpha)\lambda_t}\frac{2}{\varphi}. \tag{52}$$

The dynamics of capital is still described by equations (37) and (38); in Appendix 5 I report the steady state solutions for this economy. With respect to the previous section, here I want to analyze the effect of a shock to the realized nominal interest rate R_t^N. From the binding borrowing constraint, we can express the nominal return R_t^N of the secured notes as a function of the natural level $\bar{R}_t$, the factors' income gap $\left(\frac{\lambda_t Y_t}{\frac{\eta-1}{\eta}\bar{Y}_t}\right)$ and inflation $\left(\frac{P_t}{P_{t-1}}\right)$:

$$R_t^N = R_t\left(\frac{P_t}{P_{t-1}}\right) = \bar{R}_t\left(\frac{\lambda_t Y_t}{\frac{\eta-1}{\eta}\bar{Y}_t}\right)\left(\frac{P_t}{P_{t-1}}\right). \tag{53}$$

In Appendix 6 I describe how to obtain the formula above. In the previous section, a shock to the interest rate was associated with the start or end of a pyramid scheme. These shocks implied a change in the natural rate $\bar{R}_t$. Here, an unexpected shock to R_t^N may also be associated with a demand effect given price rigidities. A higher R_t^N can trigger an increase in demand, which can be satisfied only by a reduction in the firms' markup. This encourages the supply of labor, from equation (52). Ultimately, the rise in the real interest rate R_t would be confirmed by the relaxation of the borrowing constraints.

A positive (negative) shock to the interest rate can induce a positive (negative) output response in the short run. This may explain the correlation between prices, credit and output we observe at the time of a boom or bust. Such a demand effect vanishes in the long run when prices can adjust. A change in inflation can offset the original shock to R_t^N; or, alternatively, the higher (lower) interest rate should be supported by the rise (fall) of a bubbly scheme as described in the previous section.

The model is solved numerically as before.[38] The economy starts from a bubble-free steady state: in period 11 the nominal interest rate jumps to a higher level driven by a positive demand shock; in period 51 it drops back to its original level because of a negative demand shock. In the main simulation I assume that the agents always coordinate to equilibria with zero inflation, so that $R_t^N = R_t \; \forall \, t$.[39] This means that, after the initial unexpected shock, a change in the creation of bubbly debt must occur in order to support a different interest rate. The path for R_t^N, R_t, h_t, inflation $\pi_t = \frac{P_t}{P_{t-1}}$ and total output Y_t are plotted in Figure 13. The supply of labor only reacts in the time of the unexpected shocks. It is relevant to note that the rise in R_t^N is not immediately associated with the emergence of a rational bubble. A bubble scheme emerges later, when the demand effect dies out. When this happens, the natural rate $\bar{R}_t$ increases and the misallocation process

[38] The parameters are: $A_H = 1.8$, $A_L = 0.6$, $\varepsilon_H = 0.2$, $\varepsilon_L = 1$, $\sigma = 0.5$, $\alpha = 0.65$, $\phi = 0.25$, $e = 0.001$, $\eta = 4$.

[39] We may think that a central bank implements a monetary policy rule that targets only the inflation.

takes place: the initial output boom is inverted and the economy moves to a lower steady state. The inverse dynamics is activated after the negative shock to R_t^N.

I also plotted the alternative paths for π_t, R_t and Y_t, in the case in which, after a demand shock to the nominal interest rate R_t^N, the economy coordinates to restore the previous real interest rate R_t. In these two alternative simulations, the positive shock to R_t^N is followed by an increase in inflation, while the negative shock is followed by a deflation in prices. Both simulations help us to highlight the benefits of a higher inflation level in preventing the emergence or the continuation of a bubbly scheme.

The exercise proposed in this section provides a potential interpretation for the dynamics of output, credit, and TFP that we observed in the recent times of low inflation. While an initial boom is associated to a positive demand effect triggered by higher market returns, the following reduction in TFP is driven by the emergence of a bubble and a misallocation of factors.

4.6 Policy Implications

By way of a final contribution, this paper looks at the policy prescriptions implicit in the model. I will start by analyzing the optimal policy of a monetary authority. In the model, the central bank can be thought as an additional investor who has the power to control the nominal value of the money-like assets, which is the nominal return of the secured credit contracts. While the authority can effectively target the inflation and close the gap between the output Y_t and its natural level $\bar{Y}_t$, it cannot address a change in the natural return $\bar{R}_t$. This means that the conventional monetary instrument has no effect on bubbles. A central bank will optimally set a rule:

$$R_t^N = \bar{R}_t \left(\frac{\lambda_t Y_t}{\frac{\eta-1}{\eta} \bar{Y}_t} \right)^{\gamma_Y} \left(\frac{P_t}{P_{t-1}} \right)^{\gamma_\pi}, \tag{54}$$

with $\gamma_Y < 0$ and $\gamma_\pi < 0$.[40] In a stochastic environment we know that the absolute value of the two policy parameters need to be big enough for the rule ensuring stability and uniqueness.

In order to influence the real return $\bar{R}_t$ and the allocation of factors, the monetary authority needs additional powers otherwise it is necessary the intervention of a fiscal authority. Given the simple structure of the framework set out here, an optimal allocation of factors is one in which all credit and labor are assigned to high productive investors.[41] In this context, a planner would promote a reallocation by discouraging L-type activity. A simple route to this would be to tax the capital income of the L-type investors. In particular,

[40] Note that with respect to a traditional Taylor rule, here the monetary authority is assumed to control the nominal realized return, not the ex-ante interest rate.

[41] Since $\varepsilon_H^\alpha A_H > \varepsilon_L^\alpha A_L$, it is always $\varepsilon_H MRK_{H,t} > \varepsilon_L MRK_{L,t}$.

for a proportional tax $\tau_{L,t}$, L-type investors would prefer to lend to H-type investors if

$$(1-\tau_{L,t})\,\varepsilon_L MRK_{L,t} < R_t; \tag{55}$$

i.e., if the equilibrium interest rate was higher than the expected return from the L-type investment.[42] In the following I assume $\tau_{L,t}$ is large enough to allow the condition to be respected.

Bubbles are still possible even if high productive borrowers obtain the entire funds. In a long run steady state, the social planner would like to target the return $\bar{R}^g$ that maximizes the aggregate welfare of the economy:

$$\log\left[\frac{1}{2}\left(1-\alpha\frac{\eta-1}{\eta}\right)Y_H\left(\bar{R}^g\right)\right]+\log\left[\frac{1}{2}\bar{R}^g\left(1-\alpha\frac{\eta-1}{\eta}\right)Y_H\left(\bar{R}^g\right)\right]$$
$$+\left(1-\phi\right)\alpha\frac{\eta-1}{\eta}Y_H\left(\bar{R}^g\right)+\bar{R}^g\frac{1-\sigma}{1-\sigma\bar{R}^g}e. \tag{56}$$

The first line refers to the utility of the workers; the second line reports the utility of the H-type investors and L-type investors. Given an optimal allocation of factors, a bubbly scheme is certainly contractionary - the only effect is to crowd-out H-type capital. However, by transferring resources from younger to older agents, the consumption of the latter may increase if the economy is dynamically inefficient.

A social planner can target an optimal rate $\bar{R}^g_{t+1}$ by imposing its monopoly on the creation of bubbly notes. The planner would set a cap on the debt creation by the private sector: it must be $d_{H,t+1} \leq \frac{\phi}{R^g_{t+1}}\alpha Y_{H,t+1}\left(\bar{R}^g_{t+1}\right)\ \forall\, t$. In addition he can directly introduce bubbly notes when $\bar{R}^g_{t+1}$ is larger than the bubble-free rate R^*_{t+1}. An optimal amount of unsecured notes $d^U_{t+1}\left(\bar{R}^g_{t+1}\right)$ can be issued by a government in the form of government bonds,[43] or by a central bank in the form of bank notes. Clearly, the fraction that the planner earns as a rent should be transferred to subsidize H-type investment.

5 Conclusions

Financial crises are typically preceded by a credit boom. According to a widespread view, the costs of a crisis originates in the sudden freezing of the credit markets. Recent contributions to the literature on rational bubbles associate these fluctuations in credit to bubbly episodes. In these papers, bubbles expand output and capital by improving the allocation of funding when productive agents are financially constrained. The burst of the bubble would then lead to a recession.

The evidence, however, shows that a rapid growth in credit promotes a misallocation of resources towards

[42] Note that a policy which reallocates resources towards productive borrowers is also desirable in the absence of a bubble in the economy, as long as $e > 0$.

[43] A similar policy is suggested by Woodford (1990).

low productive industries. For example, housing and real estate sectors are the usual recipients of an increased share of capital in a credit boom. This paper shows how this phenomenon can be explained in the rational bubble framework. Here, investors with different productivities can borrow by pledging their future income as collateral or by repaying with future new debt issues. The key intuition for the misallocation result is that borrowing through unsecured debt does not require high productivity. Instead, a credit bubble favors those borrowers who have a low probability of exiting the market in the future and can maintain a long-lived scheme.

An important result of the theory is that bubbles can promote capital accumulation even if they are contractionary. Funding would be reallocated towards lower productive sectors which have a higher propensity for accumulation. This explains both the investment misallocation and the growth in capital stock that can be observed during a credit boom.

The implications of my framework stand in stark contrast to the recent literature on rational bubbles. A bubbly expansion in credit is harmful to the economy, not from the risk of a future collapse, but for the direct misallocation it provokes. Ideal policy interventions would support an optimal allocation of resources by taxing low productivity sectors of the economy. To control the emergence of a bubble, a monetary authority should retain the monopoly on the creation of bubbly notes. Caps on the debt created by private entities is an instrument to reach this goal.

References

[1] A.B. Abel, N.G. Mankiw, L.H. Summers, and R.J. Zeckhauser. Assessing dynamic efficiency: theory and evidence. *Review of Economic Studies*, 1989.

[2] E. Bartelsman, J. Haltiwanger, and S. Scarpetta. Cross-country differences in productivity: the role of allocation and selection. *American Economic Review*, 2013.

[3] C. Borio and M. Drehmann. Assessing the risk of banking crises-revisited. *BIS Quarterly Review*, 2009.

[4] C. Borio, E. Kharroubi, C. Upper, and F. Zampolli. Labour reallocation and productivity dynamics: financial causes, real consequences. *BIS Working Papers*, 2016.

[5] S.G. Cecchetti and E. Kharroubi. Why does financial sector growth crowd out real economic growth? *BIS Working Papers*, 2015.

[6] T.V. Dang, G. Gorton, B. Holmstrom, and G. Ordonez. Banks as secret keepers. *Revise and Resubmit, American Economic Review*, 2016.

[7] D.W. Diamond and P.H. Dybvig. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 1983.

[8] P.A. Diamond. National debt in a neoclassical growth model. *American Economic Review*, 1965.

[9] E. Farhi and J. Tirole. Bubbly liquidity. *Review of Economic Studies*, 2011.

[10] F. Geerolf. Reassessing dynamic efficiency. *manuscript, Toulouse School of Economics*, 2013.

[11] M. Gertler and N. Kiyotaki. Banking, liquidity, and bank runs in an infinite horizon economy. *American Economic Review*, 2015.

[12] G. Gopinath, S. Kalemli-Ozcan, L. Karabarbounis, and C. Villegas-Sanchez. Capital allocation and productivity in south europe. *CEPR Discussion Paper*, 2015.

[13] G. Gorton and G. Ordonez. Collateral crises. *American Economic Review*, 2014.

[14] C.-T. Hsieh and P. Klenow. Misallocation and manufacturing tfp in china and india. *Quarterly Journal of Economics*, 2009.

[15] N. Kocherlakota. Bursting bubbles: consequences and cures. *mimeo, Federal Reserve Bank of Minneapolis*, 2009.

[16] A. Krishnamurthy and A. Vissing-Jorgensen. The impact of treasury supply on financial sector lending and stability. *Journal of Financial Economics*, 2015.

[17] A. Martin and J. Ventura. Economic growth with bubbles. *American Economic Review*, 2012.

[18] A. Martin and J. Ventura. Managing credit bubbles. *Journal of the European Economic Association*, Forthcoming.

[19] J. Miao and P. Wang. Bubbles and total factor productivity. *American Economic Review: Papers & Proceedings*, 2012.

[20] J. Miao and P. Wang. Sectorial bubbles, misallocation and endogenous growth. *Journal of Mathematical Economics*, 2014.

[21] G.S. Olley and A. Pakes. The dynamics of productivity in the telecommunications equipment industry. *Econometrica*, 1996.

[22] M. O'Mahony and M. P. Timmer. Output, input and productivity measures at the industry level: the eu klems database. *Economic Journal*, 2009.

[23] C.M. Reinhart and K.S. Rogoff. From financial crash to debt crisis. *American Economic Review*, 2011.

[24] D. Restuccia and R. Rogerson. Policy distortions and aggregate productivity with heterogeneous plants. *Review of Economic Dynamics*, 2008.

[25] M. Schularick and A.M. Taylor. Credit boom gone bust: monetary policy, leverage cycles and financial crises, 1870-2008. *American Economic Review*, 2012.

[26] J. Tirole. Asset bubbles and overlapping generations. *Econometrica*, 1985.

[27] M. Woodford. Public debt as private liquidity. *American Economic Review*, 1990.

Table 1

The Effect of the Growth of Credit to the Private Non-Financial Sector on Factors' Allocation when Industry's TFP Growth are Country Specific

	Value Added Growth		Capital Growth		Employment Growth		Hours worked growth		Debt-to-Equity Ratio Growth	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
TFP Growth	0.883***	1.381***	0.133	0.309	0.156***	0.332**	0.160***	0.267*	0.518**	1.305**
	(0.060)	(0.161)	(0.090)	(0.189)	(0.054)	(0.151)	(0.057)	(0.151)	(0.263)	(0.665)
square_TFP Growth	0.001	0.002	0.005	0.030**	0.004**	-0.009	0.004	-0.008	-0.001	0.018
	(0.002)	(0.007)	(0.003)	(0.013)	(0.002)	(0.008)	(0.002)	(0.012)	(0.009)	(0.052)
Interaction (TFP Growth X Credit Growth PNFS)	-0.032***	-0.036***	-0.043***	-0.042***	-0.029***	-0.035***	-0.031***	-0.031***	-0.071**	-0.072**
	(0.008)	(0.008)	(0.015)	(0.015)	(0.007)	(0.007)	(0.008)	(0.008)	(0.035)	(0.036)
Interaction (square_TFP Growth X Credit Growth PNFS)	0.001***	0.001*	-0.001**	-0.000	-0.001**	-0.001***	-0.001***	-0.001***	0.000	0.001
	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.002)
Interaction (TFP Growth X Credit PNFS to GDP in 2001)		-0.003***		-0.001		-0.001		-0.001		-0.005
		(0.001)		(0.001)		(0.001)		(0.001)		(0.004)
Interaction (square_TFP Growth X Credit PNFS to GDP in 2001)		-0.000		-0.000*		0.000*		0.000		-0.000
		(0.000)		(0.000)		(0.000)		(0.000)		(0.000)
Industry's share of total Value Added in 2001	0.266**	0.273**								
	(0.124)	(0.126)								
Industry's share of total Capital in 2001			-0.046*	-0.046*						
			(0.025)	(0.025)						
Industry's share of total Employment in 2001					-0.024	-0.026				
					(0.055)	(0.055)				
Industry's share of total Hours Worked in 2001							-0.025	-0.024		
							(0.059)	(0.059)		
Debt-to-Equity Ratio in 2001									-0.324	-0.248
									(0.601)	(0.606)
Average Asset Growth of Compustat Companies									-0.109***	-0.109***
									(0.032)	(0.032)
Number of observations	384	384	284	284	379	379	378	378	312	312
R2	0.829	0.835	0.653	0.659	0.771	0.775	0.756	0.757	0.272	0.276

Notes: Data on credit are from the "Total credit to the non-financial sector" database by the Bank for International Settlements. Data on industry growth and productivity are from the "EU KLEMS" database by the Groningen Growth and Development Center. Debt-to-Equity ratios are average across companies computed from Compustat Global and North America.

Table 2

The Effect of the Growth of Credit to Non-Financial Corporations on Factors' Allocation when Industry's TFP Growth are Country Specific

	Value Added Growth		Capital Growth		Employment Growth		Hours worked growth		Debt-to-Equity Ratio Growth	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
TFP Growth	0.804***	1.364***	0.025	0.076	0.083*	0.151	0.097**	0.137	0.442**	0.212
	(0.050)	(0.118)	(0.074)	(0.176)	(0.044)	(0.110)	(0.047)	(0.116)	(0.222)	(0.480)
square_TFP Growth	-0.003	0.006	0.002	-0.005	0.002	-0.008*	0.002	-0.009**	-0.005	0.025
	(0.002)	(0.005)	(0.003)	(0.009)	(0.002)	(0.004)	(0.002)	(0.004)	(0.008)	(0.022)
Interaction (TFP Growth X Credit Growth NF Corp.)	-0.023***	-0.060***	-0.030**	-0.031**	-0.021***	-0.034***	-0.025***	-0.032***	-0.068**	-0.036
	(0.007)	(0.010)	(0.014)	(0.014)	(0.006)	(0.009)	(0.007)	(0.009)	(0.033)	(0.041)
Interaction (square_TFP Growth X Credit Growth NF Corp.)	0.001***	0.000	-0.001	-0.001	-0.000	-0.001***	-0.001**	-0.001*	0.001	0.003
	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.003)
Interaction (TFP Growth X Credit NF Corp. to GDP in 2001)		-0.005***		-0.001		-0.001		-0.001		0.003
		(0.001)		(0.002)		(0.001)		(0.001)		(0.004)
Interaction (square_TFP Growth X Credit NF Corp. to GDP in 2001)		-0.000		0.000		0.000***		0.000***		-0.001*
		(0.000)		(0.000)		(0.000)		(0.000)		(0.000)
Industry's share of total Value Added in 2001	0.308**	0.377***								
	(0.127)	(0.123)								
Industry's share of total Capital in 2001			-0.045*	-0.046*						
			(0.026)	(0.026)						
Industry's share of total Employment in 2001					-0.022	-0.020				
					(0.055)	(0.054)				
Industry's share of total Hours Worked in 2001							-0.022	-0.018		
							(0.060)	(0.059)		
Debt-to-Equity Ratio in 2001									-0.337	-0.427
									(0.599)	(0.600)
Average Asset Growth of Compustat Companies									-0.109***	-0.104***
									(0.032)	(0.032)
Number of observations	384	384	284	284	379	379	378	378	312	312
R2	0.826	0.840	0.647	0.648	0.767	0.777	0.753	0.762	0.274	0.283

Notes: Data on credit are from the "Total credit to the non-financial sector" database by the Bank for International Settlements. Data on industry growth and productivity are from the "EU KLEMS" database by the Groningen Growth and Development Center. Debt-to-Equity ratios are average across companies computed from Compustat Global and North America.

Table 3

The Effect of the Growth of Credit to the Private Non-Financial Sector on Factors' Allocation when the US Industry's TFP Growth are Used as a Proxy

	Value Added Growth		Capital Growth		Employment Growth		Hours worked growth		Debt-to-Equity Ratio Growth	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
Interaction (TFP Growth in the US X Credit Growth PNFS)	-0.044***	-0.051***	-0.034***	-0.033***	-0.037***	-0.039***	-0.033***	-0.035***	-0.079**	-0.079**
	(0.016)	(0.016)	(0.011)	(0.011)	(0.007)	(0.007)	(0.007)	(0.007)	(0.033)	(0.034)
Interaction (square_TFP Growth in the US X Credit Growth PNFS)	0.002	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.004	0.004
	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.004)	(0.004)
Interaction (TFP Growth in the US X Credit PNFS to GDP in 2001)		-0.007***		0.001		-0.002**		-0.001		-0.000
		(0.002)		(0.001)		(0.001)		(0.001)		(0.004)
Interaction (square_TFP Growth in the US X Credit PNFS to GDP in 2001)		0.001**		-0.000*		-0.000		-0.000		0.000
		(0.000)		(0.000)		(0.000)		(0.000)		(0.000)
Industry's share of total Value Added in 2001	0.166	0.219								
	(0.255)	(0.251)								
Industry's share of total Capital in 2001			-0.046*	-0.046*						
			(0.026)	(0.026)						
Industry's share of total Employment in 2001					-0.070	-0.085				
					(0.057)	(0.057)				
Industry's share of total Hours Worked in 2001							-0.059	-0.075		
							(0.063)	(0.063)		
Debt-to-Equity Ratio in 2001									-0.475	-0.477
									(0.665)	(0.668)
Average Asset Growth of Compustat Companies									-0.115***	-0.115***
									(0.035)	(0.035)
Number of observations	348	348	255	255	346	346	345	345	284	284
R2	0.334	0.361	0.643	0.650	0.769	0.778	0.749	0.758	0.248	0.248

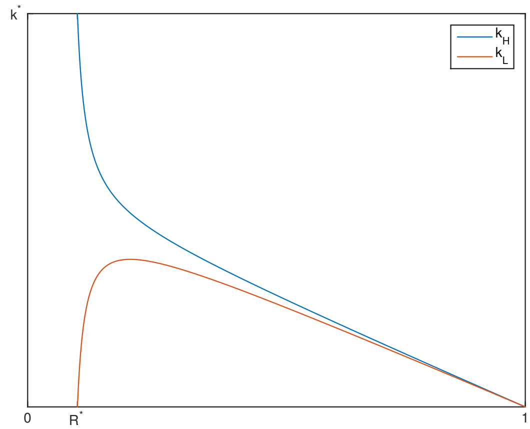
Notes: Data on credit are from the "Total credit to the non-financial sector" database by the Bank for International Settlements. Data on industry growth and productivity are from the "EU KLEMS" database by the Groningen Growth and Development Center. Debt-to-Equity ratios are average across companies computed from Compustat Global and North America.

Table 4
The Effect of the Growth of Credit to Non-Financial Corporations on Factors' Allocation when the US Industry's TFP Growth are Used as a Proxy

	Value Added Growth		Capital Growth		Employment Growth		Hours worked growth		Debt-to-Equity Ratio Growth	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
Interaction (TFP Growth in the US X Credit Growth NF Corp.)	-0.033**	-0.096***	-0.028**	-0.029**	-0.033***	-0.046***	-0.030***	-0.040***	-0.076**	-0.068*
	(0.016)	(0.018)	(0.011)	(0.011)	(0.007)	(0.008)	(0.007)	(0.009)	(0.033)	(0.037)
Interaction (square_TFP Growth in the US X Credit Growth NF Corp.)	0.002	0.007***	0.001	0.001	0.001	0.001	0.001	0.001	0.004	0.004
	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.004)	(0.004)
Interaction (TFP Growth in the US X Credit NF Corp. to GDP in 2001)		-0.013***		-0.000		-0.003***		-0.002**		0.002
		(0.002)		(0.002)		(0.001)		(0.001)		(0.005)
Interaction (square_TFP Growth in the US X Credit NF Corp. to GDP in 2001)		0.001***		-0.000		-0.000		-0.000		-0.000
		(0.000)		(0.000)		(0.000)		(0.000)		(0.001)
Industry's share of total Value Added in 2001	0.161	0.250								
	(0.256)	(0.244)								
Industry's share of total Capital in 2001			-0.046*	-0.046*						
			(0.026)	(0.026)						
Industry's share of total Employment in 2001					-0.067	-0.083				
					(0.058)	(0.057)				
Industry's share of total Hours Worked in 2001							-0.058	-0.073		
							(0.064)	(0.064)		
Debt-to-Equity Ratio in 2001									-0.502	-0.507
									(0.665)	(0.668)
Average Asset Growth of Compustat Companies									-0.115***	-0.116***
									(0.035)	(0.035)
Number of observations	348	348	255	255	346	346	345	345	284	284
R2	0.328	0.399	0.637	0.641	0.765	0.773	0.746	0.752	0.247	0.248

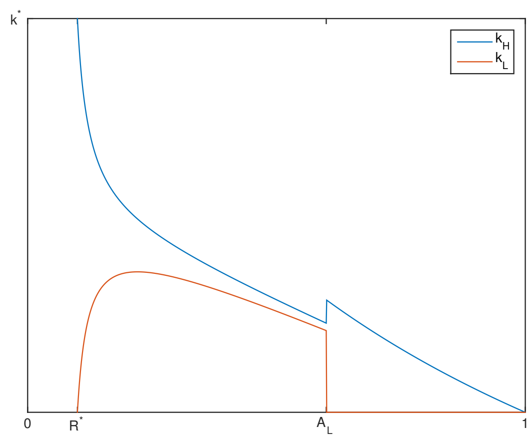
Notes: Data on credit are from the "Total credit to the non-financial sector" database by the Bank for International Settlements. Data on industry growth and productivity are from the "EU KLEMS" database by the Groningen Growth and Development Center. Debt-to-Equity ratios are average across companies computed from Compustat Global and North America.

Figure 3



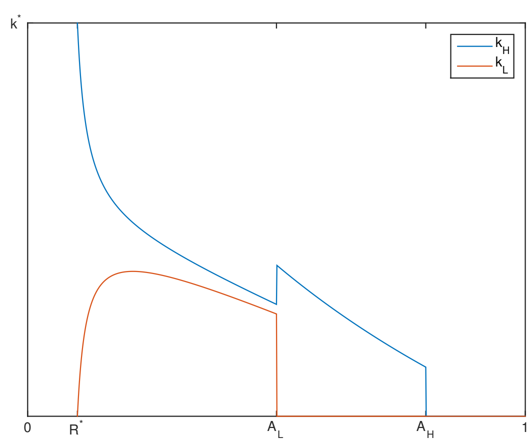
Steady state values of k_H and k_L as a function of R^b, given $\delta = 0.5$, when $1 \leq A_L < A_H$.

Figure 4



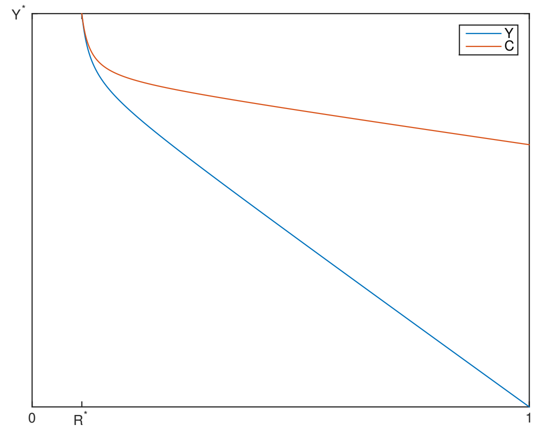
Steady state values of k_H and k_L as a function of R^b, given $\delta = 0.5$, when $A_L < 1 \leq A_H$.

Figure 5



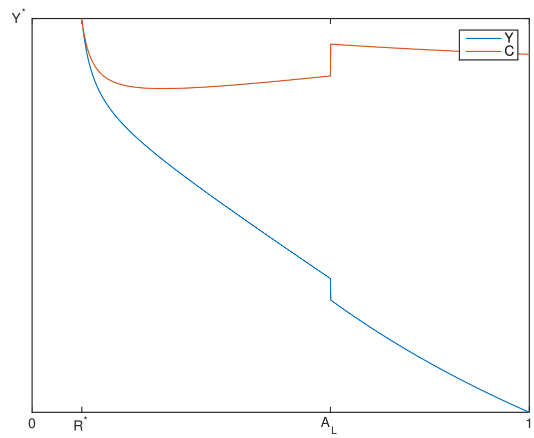
Steady state values of k_H and k_L as a function of R^b, given $\delta = 0.5$, when $A_L < A_H < 1$.

Figure 6



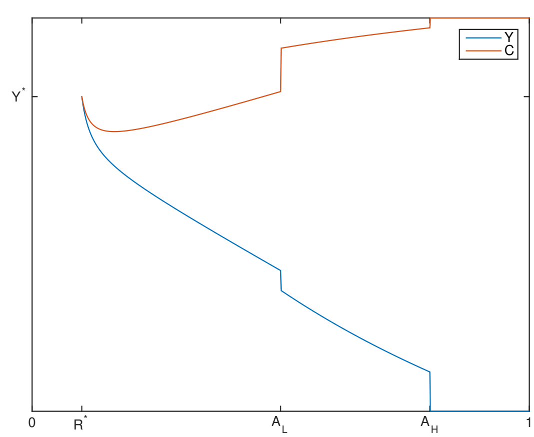
Steady state values of aggregate output and consumption as a function of R^b, given $\delta = 0.5$, when $1 \leq A_L < A_H$.

Figure 7



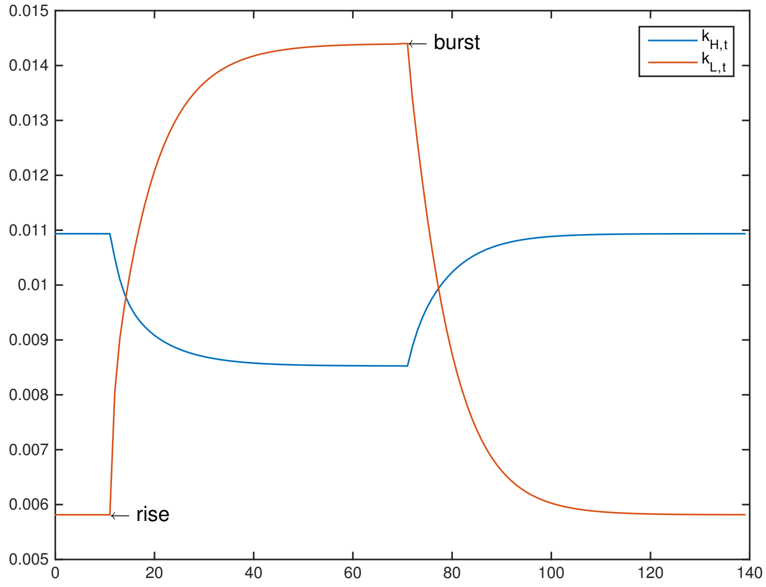
Steady state values of aggregate output and consumption as a function of R^b, given $\delta = 0.5$, when $A_L < 1 \leq A_H$.

Figure 8



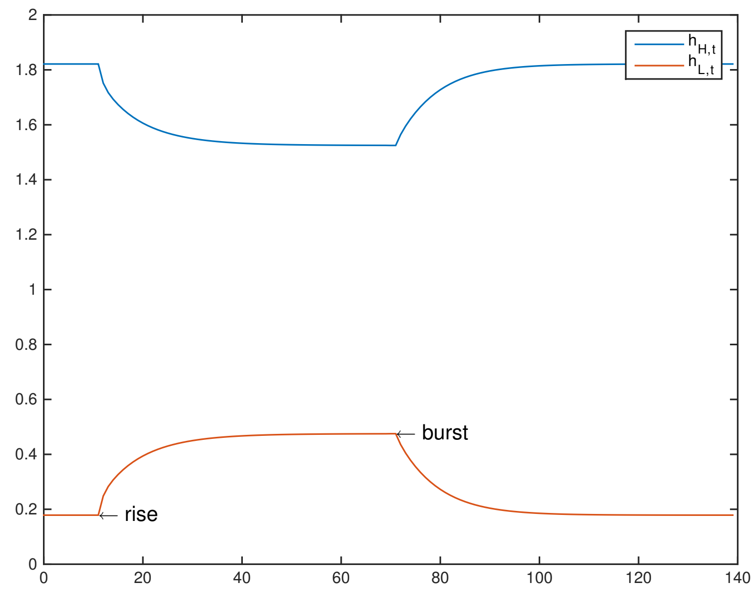
Steady state values of aggregate output and consumption as a function of R^b, given $\delta = 0.5$, when $A_L < A_H < 1$.

Figure 9



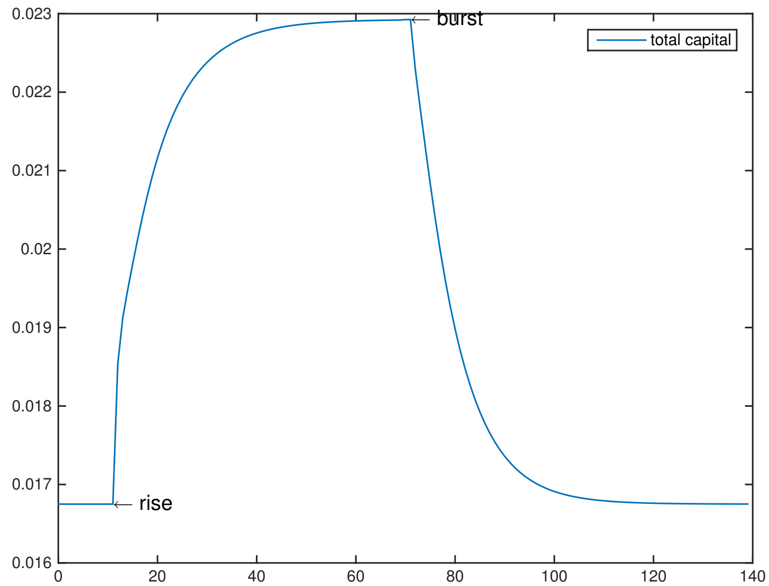
Simulation of the dynamics for $k_{H,t}$ and $k_{L,t}$. The dynamics is initiated by an unexpected positive shock to R_t at time 11 and a negative shock at time 71.

Figure 10



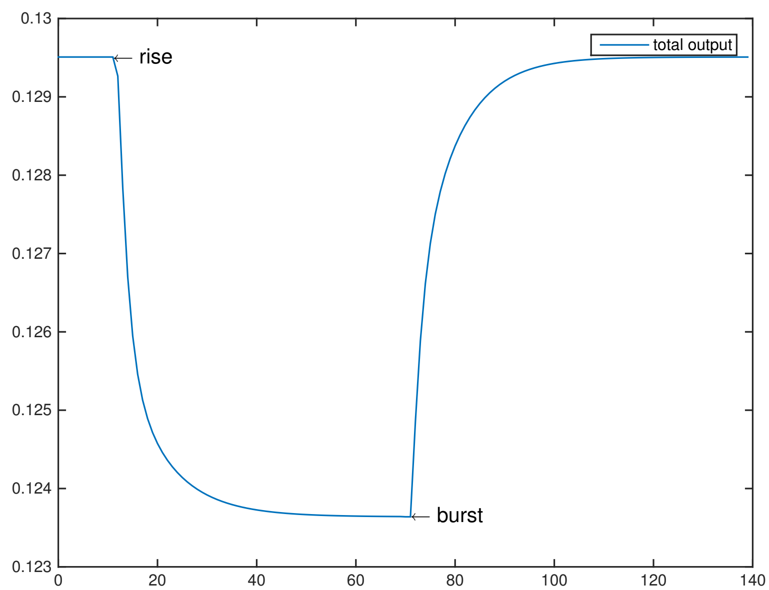
Simulation of the dynamics for $h_{H,t}$ and $h_{L,t}$. The dynamics is initiated by an unexpected positive shock to R_t at time 11 and a negative shock at time 71.

Figure 11



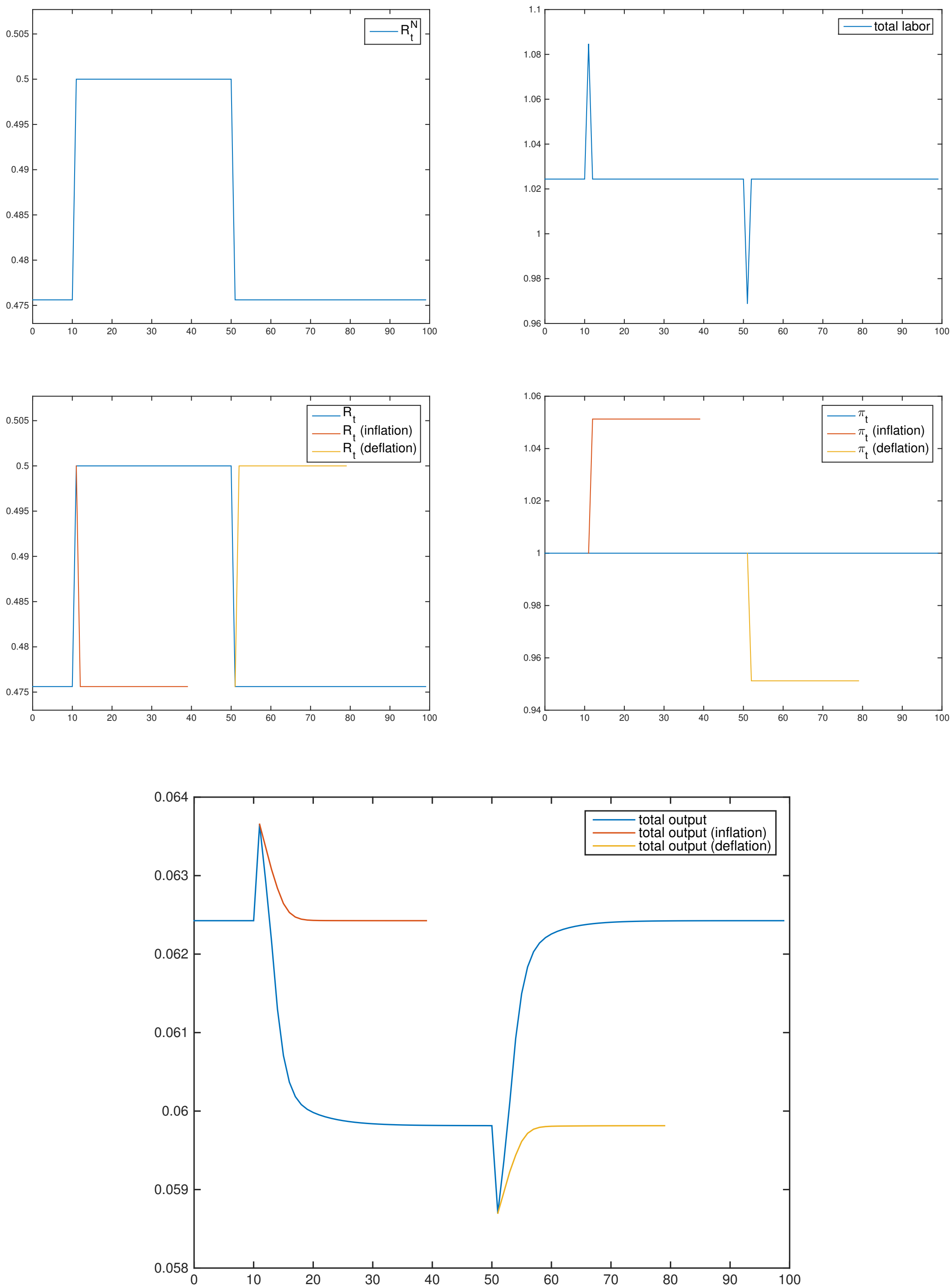
Simulation of the dynamics for the total capital $k_{H,t} + k_{L,t}$. The dynamics is initiated by an unexpected positive shock to R_t at time 11 and a negative shock at time 71.

Figure 12



Simulation of the dynamics for the total output $Y_t = Y_{H,t} + Y_{L,t}$. The dynamics is initiated by an unexpected positive shock to R_t at time 11 and a negative shock at time 71.

Figure 13



In these pictures I plot the simulation of the dynamics with nominal rigidities for R_t^N,h_t, R_t, π_t and Y_t. The dynamics is initiated by an unexpected positive shock to R_t^N at time 11 and a negative shock at time 51. Nominal rigidities play a role only in the periods of the shocks. In the main simulation (blue line) I assume a zero inflation path. In the first alternative simulation (orange line), I assume the economy restores the original real rate preceding the positive shock, by an increase in inflation. In the second alternative simulation (yellow line), I assume the economy restores the original real rate preceding the negative shock, by a reduction in inflation.

Appendix 1: Proof of Proposition 2

We will prove that aggregate capital and output is always lower if $R^b > R^*$. When it is $R^b \leq A_L$, the aggregate capital is

$$K = \frac{\chi}{\Upsilon} w =$$

$$\frac{\left(1 - \frac{\phi}{R^b} A_L + \delta \frac{1-R^b}{R^b} \phi \frac{A_L - A_H}{1-\phi A_H}\right)(1-\delta) \frac{1-R^b}{R^b} \frac{R^b - \phi A_L}{1-\phi A_L} + \left(1 - \frac{\phi}{R^b} + (1-\delta) \frac{1-R^b}{R^b} \phi \frac{A_H - A_L}{1-\phi A_L}\right) \delta \frac{1-R^b}{R^b} \frac{R^b - \phi A_H}{1-\phi A_H}}{\left(1 - \frac{\phi}{R^b} A_L + \delta \frac{1-R^b}{R^b} \phi \frac{A_L - A_H}{1-\phi A_H}\right)\left(1 - \frac{\phi}{R^b} A_H + (1-\delta) \frac{1-R^b}{R^b} \phi \frac{A_H - A_L}{1-\phi A_L}\right)} w. \tag{57}$$

The denominator is bigger than the numerator if $R^b > \phi A_H = R^*$. In fact, it is

$$\Upsilon = \chi + \left(\frac{R^b - \phi A_H}{R^b} \frac{R^b - \phi A_L}{R^b}\right)\left\{1 - \left(1 - R^b\right) \frac{1 - \phi\left[(1-\delta) A_H + \delta A_L\right]}{(1-\phi A_H)(1-\phi A_L)}\right\}, \tag{58}$$

where both the quantities in the round and curly brackets are positive. Then it must be $K < w = K^*$. When it is $A_L < R^b \leq A_H$, the aggregate capital is

$$K = k_H = \frac{1 - \delta\left(1 - R^b\right) - R^b}{1 - \delta\left(1 - R^b\right) - \phi A_H} w. \tag{59}$$

Also in this case the ratio is lower than one as long as $R^b > R^*$, and it must be $K < K^*$.

The result trivially follows for the aggregate output, given that $Y = A_H k_H + A_L k_L \leq A_H(k_H + k_L) < A_H w = Y^*$.

Appendix 2: Proof of proposition 3

We will start showing that $\frac{\partial K}{\partial \delta}$ and$\frac{\partial Y}{\partial \delta}$ are negative for both cases with $R^* < R^b \leq A_L$ and $A_L < R^b \leq A_H$. In the first case the aggregate capital K can be expressed as a function of k_H or k_L:

$$K = \frac{\left(1 - R^b\right) w + \phi\left(A_L - A_H\right) k_L}{1 - \phi A_H} = \frac{\left(1 - R^b\right) w + \phi\left(A_H - A_L\right) k_H}{1 - \phi A_L}. \tag{60}$$

Given that

$$\frac{\partial k_L}{\partial \delta} = \frac{\frac{1-R^b}{R^b} \frac{\left(R^b - \phi A_H\right)\left(R^b - \phi A_L\right)}{R^b(1-\phi A_H)}}{\left(1 - \frac{\phi}{R^b} A_L + \delta \frac{1-R^b}{R^b} \phi \frac{A_L - A_H}{1-\phi A_H}\right)^2} w > 0, \tag{61}$$

it must be $\frac{\partial K}{\partial \delta} < 0$. The result follows for the aggregate output, since it is $Y = \frac{K - \left(1-R^b\right) w}{\phi}$.

In the case of $A_L < R^b \leq A_H$ we can derive

$$\frac{\partial k_H}{\partial \delta} = \frac{\left(1-R^b\right)\left(\phi A_H - R^b\right)}{\left[1-\delta\left(1-R^b\right)-\phi A_H\right]^2} w < 0. \tag{62}$$

Then, it must be $\frac{\partial K}{\partial \delta} < 0$ and $\frac{\partial Y}{\partial \delta} < 0$, since $Y = A_H K = A_H k_H$.

The effect of an increase in δ on the aggregate consumption varies in the two cases with $R^* < R^b \leq A_L$ and $A_L < R^b \leq A_H$. Aggregate consumption in steady state is given by:

$$C = \begin{cases} A_H w & if\ R^b = R^* \\ R^b w + (1-\phi) Y & if\ R^* < R^b \leq A_L \\ R^b w + (1-\phi) Y + R^b d_L^U & if\ A_L < R^b \leq A_H \\ w & if\ R^b > A_H \end{cases} . \tag{63}$$

When $R^* < R^b \leq A_L$, it is $\frac{\partial C}{\partial \delta} = (1-\phi)\frac{\partial Y}{\partial \delta} < 0$. In the case of $A_L < R^b \leq A_H$ we can simplify the derivative:

$$\frac{\partial C}{\partial \delta} = \frac{1-R^b}{\left[1-\delta\left(1-R^b\right)-\phi A_H\right]^2}\left(R^b - \phi A_H\right)\left(1-A_H\right) w. \tag{64}$$

A higher δ increases the aggregate consumption only if $A_H < 1$.

Appendix 3: Proof of Proposition 4

We will prove that an increase in R^b always reduces the steady state wage. From labor market clearing it is:

$$\frac{(1-\sigma)e}{\left(\frac{w}{(1-\alpha)\varepsilon_H^\alpha A_H}\right)^{\frac{1}{\alpha}} - \left[\sigma(1-\phi) + \frac{\phi}{R^b}\right]\frac{\alpha}{1-\alpha} w} + \frac{(1-\sigma)e + \left(1-R^b\right)\frac{R^b - R^*}{R^b}\frac{w}{\varphi}}{\left(\frac{w}{(1-\alpha)\varepsilon_L^\alpha A_L}\right)^{\frac{1}{\alpha}} - \left[\sigma(1-\phi) + \frac{\phi}{R^b}\right]\frac{\alpha}{1-\alpha} w} = \frac{2}{\varphi}. \tag{65}$$

Taking derivatives with respect to R^b, we get:

$$-\left[\frac{\frac{R^*}{2(R^b)^2} w}{\left(\frac{w}{(1-\alpha)\varepsilon_H^\alpha A_H}\right)^{\frac{1}{\alpha}} - p\left(R^b, w\right)} - \frac{\frac{R^*}{2(R^b)^2} w}{\left(\frac{w}{(1-\alpha)\varepsilon_L^\alpha A_L}\right)^{\frac{1}{\alpha}} - p\left(R^b, w\right)}\right] h_H - \frac{\frac{w}{\varphi}}{\left(\frac{w}{(1-\alpha)\varepsilon_L^\alpha A_L}\right)^{\frac{1}{\alpha}} - p\left(R^b, w\right)} =$$

$$\left\{\frac{\left[\frac{1}{\alpha}\left(\frac{w}{(1-\alpha)\varepsilon_H^\alpha A_H}\right)^{\frac{1}{\alpha}} - p\left(R^b, w\right)\right] h_H}{\left(\frac{w}{(1-\alpha)\varepsilon_H^\alpha A_H}\right)^{\frac{1}{\alpha}} - p\left(R^b, w\right)} + \frac{\left[\frac{1}{\alpha}\left(\frac{w}{(1-\alpha)\varepsilon_L^\alpha A_L}\right)^{\frac{1}{\alpha}} - p\left(R^b, w\right)\right] h_L - q\left(R^*, R^b, w\right)}{\left(\frac{w}{(1-\alpha)\varepsilon_L^\alpha A_L}\right)^{\frac{1}{\alpha}} - p\left(R^b, w\right)}\right\} \frac{1}{w}\frac{\partial w}{\partial R^b} \tag{66}$$

with $q\left(R^*, R^b, w\right) = \left(1 - R^b\right) \frac{R^b - R^*}{R^b} \frac{w}{\varphi}$ and $p\left(R^b, w\right) = \left[\sigma\left(1 - \phi\right) + \frac{\phi}{R^b}\right] \frac{\alpha}{1-\alpha} w$. The equation always implies $\frac{\partial w}{\partial R^b} < 0$. In fact, the left hand side is negative for sure, given $\varepsilon_H^\alpha A_H > \varepsilon_L^\alpha A_L$.

Appendix 4: Proof of Proposition 5

Aggregate capital in steady state is given by:

$$K = \left(\varepsilon_H + \varepsilon_L\right)\left(1 - \sigma\right) e + \left(\varepsilon_H h_H + \varepsilon_L h_L\right)\left[\sigma\left(1 - \phi\right) + \frac{\phi}{R^b}\right] \frac{\alpha}{1 - \alpha} w + \varepsilon_L \left(1 - R^b\right) \frac{R^b - R^*}{R^b} \frac{w}{\varphi}. \tag{67}$$

Taking derivatives with respect to R^b at $R^b = R^*$ we obtain:

$$\begin{aligned}\frac{\partial K}{\partial R^b} = \varepsilon_L &\left\{\left[\sigma\left(1 - \phi\right) + \frac{\phi}{R^*}\right] \frac{\alpha}{1 - \alpha}\left[h_L^* \frac{\partial w}{\partial R^b} - w^* \frac{\partial h_H}{\partial R^b}\right] + \left[\frac{1 - R^*}{\varphi R^*} - \frac{1}{2R^*} h_L^*\right] w^*\right\} \\ &+ \varepsilon_H \left\{\left[\sigma\left(1 - \phi\right) + \frac{\phi}{R^*}\right] \frac{\alpha}{1 - \alpha}\left[h_H^* \frac{\partial w}{\partial R^b} + w^* \frac{\partial h_H}{\partial R^b}\right] - \frac{1}{2R^*} h_H^* w^*\right\}.\end{aligned} \tag{68}$$

It is easy to see that $\frac{\partial K}{\partial R^b}$ would certainly be negative if $\varepsilon_H = \varepsilon_L$.

In what follows we will show that $\frac{\partial K}{\partial R^b}$ would be positive for small ϕ and ε_H. For simplicity, we will prove the claim for the case $\varepsilon_H^\alpha A_H = \varepsilon_L^\alpha A_L$. We start by taking the limit of $\frac{\partial K}{\partial R^b}$ for ε_H approaching 0 given a constant $\rho = \varepsilon_H^\alpha A_H = \varepsilon_L^\alpha A_L$, i.e. by assuming $A_H = A_H\left(\varepsilon_H\right) = \frac{\rho}{\varepsilon_H^\alpha}$. Note that, as soon as $\varepsilon_H^\alpha A_H$ and $\varepsilon_L^\alpha A_L$ are constant, h_H^*, h_L^*, w^*, $\frac{\partial w}{\partial R^b}$ and $\frac{\partial h_H}{\partial R^b}$ do not change as ε_H goes to 0. Then it will be:

$$\lim_{\varepsilon_H \to 0} \frac{\partial K}{\partial R^b} = \varepsilon_L \left\{\left[\sigma\left(1 - \phi\right) + \frac{1 - \alpha}{\alpha}\right] \frac{\alpha}{1 - \alpha}\left[\frac{1}{\varphi} \frac{\partial w}{\partial R^b} - w^* \frac{\partial h_H}{\partial R^b}\right] + \left[\frac{1 - 2R^*}{2\varphi R^*}\right] w^*\right\}. \tag{69}$$

$\left[\frac{1-2R^*}{2\varphi R^*}\right]$ would be positive for a small R^*. Moreover, we can rewrite $\left[\frac{1}{\varphi} \frac{\partial w}{\partial R^b} - w^* \frac{\partial h_H}{\partial R^b}\right]$ as:

$$\frac{\left(w^*\right)^2}{2\varphi}\left[\frac{\frac{1+R^*}{R^*}}{\left(\frac{w^*}{(1-\alpha)\varepsilon_H^\alpha A_H}\right)^{\frac{1}{\alpha}} - \left[\sigma\left(1 - \phi\right) + \frac{\phi}{R^*}\right] \frac{\alpha}{1-\alpha} w^*} - \frac{1}{\frac{1}{\alpha}\left(\frac{w^*}{(1-\alpha)\varepsilon_H^\alpha A_H}\right)^{\frac{1}{\alpha}} - \left[\sigma\left(1 - \phi\right) + \frac{\phi}{R^*}\right] \frac{\alpha}{1-\alpha} w^*}\right], \tag{70}$$

where the quantity in the square brackets is always positive.

Appendix 5: Steady State of Model with Nominal Rigidities

In steady state, nominal rigidities cannot play any role. If firms can optimally set their price, it must be:

$$h_t = \frac{(\eta - 1)(1 - \alpha)}{1 + (\eta - 1)(1 - \alpha)} \frac{2}{\varphi}. \tag{71}$$

Without bubbles, the equilibrium interest rate is pinned down from the binding borrowing constraint:

$$R^* = 2\frac{(\eta - 1)(1 - \alpha)}{1 + (\eta - 1)(1 - \alpha)} \phi \frac{\alpha}{1 - \alpha}. \tag{72}$$

The steady state capital allocation is:

$$k_H = \varepsilon_H \left\{ (1 - \sigma) e + \left[\sigma (1 - \phi) + \frac{\phi}{R^b} \right] \frac{\alpha}{1 - \alpha} w h_H \right\} \tag{73}$$

$$k_L = \varepsilon_L \left\{ (1 - \sigma) e + \left[\sigma (1 - \phi) + \frac{\phi}{R^b} \right] \frac{\alpha}{1 - \alpha} w h_L + \left(1 - R^b\right) \left[\frac{1}{R^*} - \frac{1}{R^b} \right] \phi \frac{\alpha}{1 - \alpha} w h \right\}. \tag{74}$$

The steady state labor allocation as a function of w and aggregate labor supply h is:

$$h_H = \frac{(1 - \sigma) e}{\left(\frac{\frac{\eta}{\eta - 1} w}{(1 - \alpha) \varepsilon_H^\alpha A_H} \right)^{\frac{1}{\alpha}} - \left[\sigma (1 - \phi) + \frac{\phi}{R^b} \right] \frac{\alpha}{1 - \alpha} w} \tag{75}$$

$$h_L = \frac{(1 - \sigma) e + \left(1 - R^b\right) \left[\frac{1}{R^*} - \frac{1}{R^b} \right] \phi \frac{\alpha}{1 - \alpha} w h}{\left(\frac{\frac{\eta}{\eta - 1} w}{(1 - \alpha) \varepsilon_L^\alpha A_L} \right)^{\frac{1}{\alpha}} - \left[\sigma (1 - \phi) + \frac{\phi}{R^b} \right] \frac{\alpha}{1 - \alpha} w}. \tag{76}$$

We can finally solve for the steady state w from the labor market clearing: $h_H + h_L = \frac{(\eta - 1)(1 - \alpha)}{1 + (\eta - 1)(1 - \alpha)} \frac{2}{\varphi}$.

Appendix 6

From the binding borrowing constraint we can find the real return at time t:

$$R_t = \frac{2}{\varphi} \phi \frac{\alpha \lambda_t}{(1 - \lambda_t) + (1 - \alpha) \lambda_t} \frac{w_t}{d_t^S}; \tag{77}$$

with flexible prices it must be:

$$\bar{R}_t = \frac{2}{\varphi} \phi \frac{\alpha (\eta - 1)}{1 + (1 - \alpha)(\eta - 1)} \frac{\bar{w}_t}{d_t^S}. \tag{78}$$

Then, we can rewrite:

$$R_t = \bar{R}_t \frac{\lambda_t w_t}{(1 - \lambda_t) + (1 - \alpha) \lambda_t} \frac{1 + (1 - \alpha)(\eta - 1)}{(\eta - 1) \bar{w}_t}. \tag{79}$$

From the equilibrium in the labor market it is:

$$w_t = \frac{\varphi}{2}\left[(1-\lambda_t) + (1-\alpha)\,\lambda_t\right] Y_t; \tag{80}$$

then, we finally obtain

$$R_t^N = R_t\left(\frac{P_t}{P_{t-1}}\right) = \bar{R}_t\left(\frac{\lambda_t Y_t}{\frac{\eta-1}{\eta}\bar{Y}_t}\right)\left(\frac{P_t}{P_{t-1}}\right). \tag{81}$$