Balance sheet policy above the effective lower bound

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Abstract

When the short rate is above the effective lower bound, a central bank does not need a large balance sheet to steer the economy. I discuss factors relevant for setting balance sheet size and composition above the ELB, focusing on the role of convenience yields. Reserves provide a liquid and safe asset suggesting that a central bank may want to supply enough to set their convenience yield of zero. However, if reserves are supplied via purchases of bonds (e.g., Treasuries or Bunds) which themselves have convenience yields, the convenience-maximizing central bank balance sheet size equalizes the convenience yields on reserves and bonds and is smaller than the former prescription would imply. I estimate the conveniencemaximizing balance sheet size for the US and the euro area, emphasizing different political constraints across the Atlantic on which types of assets central banks can hold without being perceived as affecting credit allocation or fiscal policy.

1 Introduction

The global financial crisis marked a turning point in central bank policymaking in many jurisdictions. When short-maturity policy rates reached the effective lower bound (ELB) and more policy accommodation was needed in the years following the crisis, balance sheet policy and forward guidance were used to a much larger extent than in the past. The Federal Reserve embarked on quantitative easing (QE) starting in 2008 while the ECB purchased sovereign bonds during the European sovereign debt crisis and started its quantitative easing bond purchases in 2015. The COVID crisis saw a resurgence in bond purchases, first for financial stability purposes and later for economic stimulus.

While much has been said about how and how much QE affects asset prices and the economy, much less is known about quantitative tightening (QT), both from a positive and a normative perspective. The Federal Reserve reduced its balance sheet by about \$700B between April 2017 and August 2019 but reversed course in September 2019 after a spike in various short-term market rates suggested that the reduction in reserve supply led to an undesired scarcity of liquidity. The ECB's

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balance sheet plateaued in 2019 but did not shrink. With the surge in inflation across the Atlantic, both the Federal Reserve and the ECB have raised their short policy rates substantially and have started balance sheet reduction. However, there appears to be little consensus on how much balance sheet reduction is desirable. The Federal Reserve has announced monthly balance sheet reductions of up to \$95B/month but has not announced an end-date to its balance sheet reduction.² The ECB has announced balance sheet reduction of €15B/month until the end of June 2023 with the subsequent pace to be determined over time. This is in addition to balance sheet reduction from banks' repayment of TLTRO borrowing.

Against this background, this paper asks what balance sheet size is desirable when a large balance sheet is not needed for policy stimulus, nor for policy tightening. There is no upper bound on short policy rates and while quantitative tightening contributes to tightening the overall policy stance for a given policy rate, it is wellknown that a given amount of policy restraint on the economy can be achieved by a variety of combinations of the short policy rate and balance sheet size. I start with a simple graphical exposition of this fact, illustrating what I label the "iso-market rate curve" which is combinations of the interest rate on reserves and reserve supply that achieves the same short market rate. The iso-market rate curve is derived from the demand curve for reserves. The interest rate on reserves affects the demand curve for reserves. Given the demand curve, reserve supply determines reserve scarcity, which maps to the spread between the short market rate and the interest rate on reserves. A central bank wishing to reach a given short market rate can do so with either a low interest rate on reserves, a low reserve supply, and substantial reserve scarcity or with a higher interest rate on reserves, a higher reserve supply and less (or no) reserve scarcity. I extend the iso-market rate curve concept to longer market rates that better capture the overall policy stance.

The central question I ask is: How should central banks choose a point on the isomarket rate curve? In other words, should they choose a small balance sheet or a large one? Should they target scarce or ample reserve supply? I briefly review factors that may be relevant for this choice, including (a) the central bank's supply of liquid and safe assets, (b) interest rate volatility, (c) side effects of large central bank balance sheets, and (d) central bank profit risk, independence, and headroom for future QE. My analysis then focuses on the first factor, the effect of balance sheet size on the central bank's supply of liquid and safe assets to the economy. This is not to say that only this factor matters. I view my estimates as a benchmark from which policymakers can adjust balance sheet size up or down depending on their view of the importance of other factors (which go in different directions).

The starting point for my analysis is what one could call a Friedman Rule for reserves: Supply reserves to the point that they are no longer scarce. A central bank can create reserves at little cost, and this reserve supply maximizes welfare from the liquidity value of reserves. I argue, however, that whether following this approach is desirable or not, depends on how a central bank supplies reserves. If the central bank supplies reserves via purchases of bonds that themselves have a convenience

² See Federal Reserve Board - Plans for Reducing the Size of the Federal Reserve's Balance Sheet and Monetary policy decisions (europa.eu).

yield from liquidity or safety, maximizing the central bank's net supply of convenient assets should account for both the assets supplied and the assets purchased to supply them. While a central bank able to supply assets via holdings of assets without convenience yields (I will denote this by Case A) will maximize its convenience supply by setting the convenience yield on reserves to zero, a central bank that supplies reserves via holdings of convenient assets (Case B) maximizes its net convenience supply by equalizing the convenience yield on reserves and the (average) convenience yield on assets purchased to supply the reserves. The analysis accounts for bank balance sheet costs, arguing that these should be subtracted from convenience yields for reserves when maximizing the welfare from supplying reserves.

I derive the comparative statics resulting from convenience-maximization to emphasize that the convenience-maximizing reserve supply evolves over time. It is increasing in bank deposits because higher deposits shift the reserve demand curve outward as banks value reserves more when they have more deposits to manage. If reserves are supplied via central bank holdings of convenient assets, the convenience-maximizing reserve supply is furthermore affected by drivers of the convenience yield on these assets. These drivers include any shifters of the demand curve for the convenient assets (the size of the economy, which I proxy by GDP) as well as the asset supply and the central bank's asset holdings due to its autonomous factors (currency, government deposits etc.).

In my framework, central bank asset mix takes a central role for determining convenience-maximizing balance sheet size because a central bank's holdings of assets with a convenience yield reduces its net supply of convenience. There are sharp contrasts between the legal and political constraints on asset choice of the Federal Reserve and the ECB.

In the US, the Federal Reserve Act grants the Federal Reserve the right to buy and sell obligations that are direct obligations of, or guaranteed by, the United States. The Federal Reserve thus cannot in general transact in non-government guaranteed securities. When it purchased corporate bonds during the COVID crisis, this was done using an emergency lending program set up under Section 13-3 of the Federal Reserve Act to address "unusual and exigent circumstances". Similarly, the discount window (in which the Federal Reserve lends reserves to banks against collateral) is priced to be used mainly in crisis. Broaddus and Goodfriend (2001) express the common sentiment in the US that the Federal Reserve should continue to focus on holding Treasuries to avoid interfering with credit allocation. They write:

"...the Fed's asset acquisition policy ought to give priority to preserving public support for the Fed's independence by insulating the central bank as much as possible from potentially damaging disputes regarding credit allocation"

"When the Fed purchases Treasury securities, it extends Federal Reserve credit to the Treasury. Doing so, however, leaves all the fiscal decisions to Congress and the Treasury and hence does not infringe on their fiscal policy prerogatives. When the Fed extends credit to private or other public entities, however, it is allocating credit to particular borrowers, and therefore taking a fiscal action and invading the territory of the fiscal authorities."

The FOMC has stated its intent that the Federal Reserve will primarily hold Treasuries in the longer run "thereby minimizing the effect of Federal Reserve holdings on the allocation of credit across sectors of the economy".³

By contrast, the ECB has historically supplied reserves via collateralized lending to banks. In the euro area, ECB purchases of government bonds have been politically sensitive with various ECB programs challenged in court. The ECB only initiated large-scale asset purchases focused on government bonds in March 2015 (under the Public Sector Purchase Programme which accounted for the largest share of purchases under its Asset Purchase Programme). This was much later than the Federal Reserve and several other central banks. Schnabel (2023a) states:

"In the euro area, however, there are [...] additional considerations relevant for the assessment of whether a large bond portfolio is desirable or not. One is that the lack of a consolidated public sector balance sheet raises more fundamental concerns about monetary and fiscal interactions in a currency union with sovereign member states. These concerns may potentially undermine the credibility and independence of the central bank."

As these quotes illustrate, on both sides of the Atlantic, the central bank's assets are viewed as having potential implications for central bank independence. However, what is considered politically sensitive differs, with government bonds a politically safe choice in the US, but a politically risky choice in the euro area. From the perspective of convenience-maximization, the ECB is therefore at an advantage. ECB loans to banks can be collateralized by a range of assets, including assets without convenience yields. In practice, bank borrowing from the ECB is predominantly not backed by central government securities and market participants assess that it is unlikely that many of the securities posted are Bunds.⁴ Therefore, in the longer run the ECB would likely be able to return to providing reserves without holdings of convenient assets (and without requiring convenient assets as collateral for lending). By contrast, the high convenience yield on US Treasury securities relative to highly rated corporate bonds is well-documented (e.g., Krishnamurthy and Vissing-Jorgensen (2012), KVJ in what follows). This puts the Federal Reserve in a more difficult position from the perspective of being able to provide liquid and safe assets, on net.

With that observation, I turn to estimating convenience-maximizing liquidity supply for both the US and the euro area. In both cases, estimation of the reserve demand function is crucial. I build on recent work by Lopez-Salido and Vissing-Jorgensen (2023, LS-VJ). They emphasize three drivers of reserve demand. First, the interest

³ Federal Reserve Board - Principles for Reducing the Size of the Federal Reserve's Balance Sheet

⁴ See Eurosystem Collateral Data (europa.eu) and The ECB's collateral conundrum - Central Banking. Schnabel (2023b) provides evidence suggesting that most collateral posted by banks for ECB loans to banks in the TLTROs is rated below Credit Quality Step 1, and thus unlikely to have substantial convenience yields. ECB reserve supply via loans thus does not reduce the availability of convenient asset to the private sector the way ECB purchases of government bonds with convenience yields would.

rate spread between the short market rate and the interest rate on reserves. Second, the banking sector's need for liquid assets to manage their liquidity promises. This depends on the amount of bank deposits. Third, bank balance sheet costs (notably capital requirements such as the Supplementary Leverage Ratio (SLR)), which make it costly for banks to borrow to fund reserve holdings. A central finding of their work is that, with the control for deposits, a stable reserve demand curve emerges over the post-GFC period for the US. I update their estimates to April 2023 and extend reserve demand estimation to the euro area, finding a good fit for the period after the European sovereign debt crisis.

In the case of the US, the other input needed to calculate the conveniencemaximizing reserve supply is an estimate of the demand function for Treasuries, i.e., the convenience yield function for US Treasuries. I update the estimation of this relation from KVJ, focusing on the private (i.e., non-Federal Reserve) sector's Treasury demand. Despite the high Treasury supply in recent years, I estimate that Treasuries still carry a convenience yield. The Aaa corporate – Treasury yield spread in April 2023 averaged 66 bps, of which I estimate that around 31 bps is due to default risk in the corporate bonds (or tax differences between types of bonds), leaving a convenience-yield around 35 bps. Since the GFC, Federal Reserve Treasury purchases have reduced the net supply available to the private sector and the private sector's Treasury demand curve has shifted to the right due to Treasury purchases by foreigners, thus making Treasuries scarce even at the current high Debt/GDP ratio.

With these inputs, I estimate convenience-maximizing reserve supplies as of April 2023. For the Federal Reserve, I estimate that, given current deposits, the convenience-maximizing supply of reserves (including holdings in the overnight-reverse repurchase facility) would currently be around \$3.3T if the Federal Reserve could supply reserves with holdings of inconvenient assets. However, the convenience-maximizing reserve supply is below \$1T with reserves supplied with holdings of Treasuries, as the Federal Reserve has announced is its intention in the longer run. For the ECB, I estimate a convenience-maximizing reserve supply currently of €1.4T, with reserves supplied via inconvenient assets. Careful modelling of reserve demand is crucial in both jurisdictions. Deposits have trended upward (even before COVID). I provide time series estimates of how convenience-maximizing reserve supplies have evolved over time.

It is important to emphasize that overall welfare from convenient assets is maximized by convenience-maximization, not by maximizing the central bank's seigniorage from issuing liabilities with low interest rates due to their convenience yields. Welfare from reserves is the consumers' plus producers' surplus from reserves while the central bank's seigniorage is the producers' surplus only. Vissing-Jorgensen (2023) clarifies the difference between convenience-maximization and maximizing seigniorage from convenience yields. If reserves are supplied via holdings of inconvenient assets, the seigniorage from reserves is zero at the welfare maximizing reserve supply. By contrast, the central bank's seigniorage from reserves would be positive at a lower reserve supply. If reserves are supplied via central bank holdings of government bonds with convenience yields, then the convenience-maximizing reserve supply can be above or below that which maximizes the consolidated government's overall seigniorage from reserves and government bonds. However, even a taxpayer narrowly focused on seigniorage would do better maximizing convenience and taxing the consumer's surplus from convenient assets (if possible) than maximizing seigniorage. This emphasizes the importance of convenience-maximization.

Too many tools above the ELB: Iso-market rate curves

2

It has been well known since at least Goodfriend (2002), that if a central bank can pay interest on reserves, it can achieve the same equilibrium market rate in various ways.⁵ Figure 1, Panel a illustrates this idea. The left figure graphs reserve demand in blue and reserve supply in red. Two points, *A* and *B*, are illustrated at which the equilibrium short market rate equals the central bank's target rate. One possibility, point 1, is to set a low interest rate on reserves IOR_1 , resulting in a low reserve demand curve D_1 , and at the same time supply a modest quantity of reserves, S_1 . Another possibility, point 2, is to set a higher interest rate on reserves IOR_2 , resulting in a higher reserve demand curve D_2 , and at the same time supply a larger quantity of reserves, S_2 . These are just two possible combinations that result in the market rate clearing at the desired target level. The right figure graphs in blue all possible combinations of reserve supply and the interest rate on reserves which achieve the same target. I will refer to this schedule as the "iso-market rate curve".

The reserve demand curves in Panel a are simplified. In practice, reserve demand is not linear above the saturation point. Furthermore, in recent years it is not uncommon to observe a market rate below the interest rate on reserves in countries with large balance sheets. The standard interpretation of this is that banks must face balance sheet costs (for economic and/or regulatory reasons) when expanding their balance sheet. Lopez-Salido and Vissing-Jorgensen (2023) provide a simple framework for reserve demand that accounts for its key drivers. The reserve demand curve emerges from banks' first-order condition for borrowing to invest in reserves. Their framework has three main ingredients. First, reserves pay interest. Second, reserves have liquidity benefits for banks, captured by a convenience yield function v(Reserves, Deposits). This function captures expected transaction costs savings due to not having to sell bonds/loans when faced with deposits outflows. v(Reserves, Deposits) is positive and increasing in reserves but decreasing in deposits as more deposits imply higher, not lower, expected transactions costs. The marginal value of additional reserves, $v'_R(Reserves, Deposits)$, is the derivative of v(Reserves, Deposits) with respect to reserves. In general, I will denote the marginal convenience values by "convenience yields", since the marginal convenience values are what drives equilibrium yield discounts on convenient assets. The convenience yield $v'_{R}(Reserves, Deposits)$ is decreasing in reserves (as additional reserves are

⁵ See also Keister, Martin and McAndrews (2008) for a clear exposition of these ideas.

Figure 1



a) Simple setting with linear demand and no balance sheet costs



b) More realistic setting with curved demand and balance sheet costs





Reserve demand and supply







c) Iso-market rate curve for long market rate





less and less valuable for given deposits) and increasing in deposits (as additional reserves are more valuable when managing additional deposits).⁶ ⁷ Third, banks face a balance sheet cost φ per dollar of assets (I will discuss this cost more below). Given these ingredients, banks' first-order condition for borrowing at the short market rate r and investing in reserves at the interest rate on reserves (IOR) is

$$r = IOR + v'_R(Reserves, Deposits) - \varphi.$$
(1)

This says that the highest interest rate, r, a bank is willing to pay to borrow to invest in reserves is the net benefit obtained from earning the IOR plus the marginal liquidity benefit from additional reserves, $v'_R(Reserves, Deposits)$, minus the additional balance sheet cost incurred, φ .⁸

Equation (1) is the reserve demand curve illustrated in Figure 1, Panel b, top figure. The reserves demand curve is negatively sloped because $v'_R(Reserves, Deposits)$ is declining in the quantity of reserves held. It shifts up for a higher IOR and down for a higher φ . Once reserve demand is satiated, $v'_R(Reserves, Deposits)$ is zero and the reserve demand curve becomes flat at $IOR - \varphi$. I omit this region of the graph for simplicity.⁹

Reserve supply now determines reserve scarcity net of balance sheet costs in equilibrium. In Figure 1, Panel b, bottom two graphs, points 1 and 2 again illustrate possible ways to achieve a given target rate r, with point 1 illustrating a low *IOR* and a small balance sheet and point 2 illustrating a higher *IOR* and a larger balance sheet. The iso-market rate curve is now curved. From (1), it follows that the iso-market rate curve for a given target rate, r^{Target} , is given by

$$IOR = r^{Target} - [v_R'(Reserves, Deposits) - \varphi].$$
⁽²⁾

For a given value of reserve supply, (2) shows how to shade/increase the *IOR* relative to the target rate in order to make the market equilibrium rate clear at the target. At point 1 in Figure 1, Panel b (bottom left and right figures), reserves are sufficiently scarce (i.e., the expected marginal liquidity value of reserves sufficiently high) that $v'_R(Reserves, Deposits) - \varphi > 0$. The central bank therefore needs to set the IOR below the target to achieve it. Conversely, at point 2, reserves are so

⁶ The measure of reserves that enters v(Reserves, Deposits) is excess reserves, i.e., reserves minus required reserves. Because reserves cannot go below required reserves, required reserves cannot be used to manage deposits outflows (except to the extent that required reserves go down with deposits).

⁷ Pre-GFC, reserve requirements were binding in both the US and euro area, generating a regulatory link from deposits to reserve demand. Post-GFC, reserve requirements are typically not binding and the effect of deposits on reserve demand is due to banks' "economic" demand for reserves as laid out in equation (1).

⁸ Equation (1) assumes competitive markets. An important direction for future work is to incorporate market power by banks or lenders in the market for short-term borrowing into my estimations below. For a theory of Nash bargaining in the federal funds market, see Afonso and Lagos (2015).

⁹ This reserve demand curve shows banks' reserve demand absent any additional central bank facilities. If the central bank has a lending facility (the discount window in the US, the marginal lending facility in the euro area), the rate on this facility will (absent stigma) serve as an upper bound on the equilibrium market rate and the reserve demand curve is cut off at the point it hits the ceiling rate. Similarly, if the central bank has an investment facility accessible to non-banks (the overnight reverse repo facility in the US), the rate on that facility will be a lower bound on the equilibrium market rate and the reserve demand curve is cut off at the point it hits the ceiling rate. Similarly, if the central bank has an investment facility accessible to non-banks (the overnight reverse repo facility in the US), the rate on that facility will be a lower bound on the equilibrium market rate and the reserve demand curve is cut off at the point it hits floor rate. See Lopez-Salido and Vissing-Jorgensen (2023) for further discussion of how borrowing/lending actions by the private sector changes the equilibrium quantity of reserves to ensure that the market rate stays in the corridor between the floor and ceiling rates.

plentiful that $v'_R(Reserves, Deposits) - \varphi < 0$ and the central bank needs to set the *IOR* above the target to achieve it.

While the equilibrium short rate r equals the target along all points on the iso-market rate curve, reserves and thus balance sheet size differs. In that sense, the overall policy stance is not constant along the iso-market rate curve. If a central bank buys assets with risk (duration risk, pre-payment risk or credit risk), the spread between the rate on those assets and the short market rate will decline in balance sheet size through a host of channels explored in the literature on quantitative easing. This does not change the conclusion that a central bank can achieve a given level of policy tightness with various combinations of reserves and *IOR*. It does, however, lead to a steeper iso-market rate curve for long than short rates. I illustrate this in Figure 1, Panel c. At a smaller reserve supply, both the r - IOR spread for the short rate r and the long-short spread is larger. To hit a given target value for its overall policy stance, as summarized by the long rate, the central bank needs to set a substantially lower IOR than would be needed at with a larger reserve supply.

In sum, whether focused on long or short rates, when a central bank is not constrained by the ELB, the same level of policy stance can be achieved with various combinations of the interest rate on reserves and the size of the balance sheet. This is the case even in a realistic setting with convex reserve demand (as suggested by the data in the estimation below) and bank balance sheet costs.

3

Factors relevant for choosing balance sheet size above the ELB

What differs along the iso-market rate curve (for the long market rate) that may guide optimal balance sheet size and composition when the short rate is above the ELB and the ELB is thus not binding?

a) Central bank liabilities: Liquidity/safety supply

From a consolidated government perspective, a larger central bank balance sheet funded with reserves leads to an overall shortening of the duration of the consolidated government's debt. As reviewed by Greenwood, Hanson, Rudolph and Summers (2016), government debt maturity structure does not matter for household resources or welfare unless some of the assumptions underlying Ricardian equivalence fails. Ricardian equivalence assumes away convenience yields (and assumes that taxation is non-distortionary). A larger central bank balance sheet then simply shortens the consolidated government's debt maturity and has no effect on welfare.¹⁰ My analysis will focus on convenience yields, emphasizing that they can

¹⁰ Greenwood et al (2016) state the intuition well: "Consider a government with an initial accumulated deficit and no future expenditures that must decide whether to finance its deficit by issuing short- or long-term bonds. If the government finances itself solely through the issuance of short- term debt, then the government will have to raise taxes if short-term interest rates rise. However, the rise in interest rates will leave a household that is lending short-term to the government with a bit more in its bank account. Since the government's sources of funds (taxes and proceeds from issuing new debt) must equal its uses of funds (paying off maturing debt), the gain in the house hold's bank accounts must precisely offset the increase in taxes."

be (and in practice are) different for reserves, bills and bonds, and laying out the implication this has for the welfare-maximizing central bank balance sheet size.

With convenience yields, a larger central bank balance sheet is not simply a welfareneutral shortening of consolidated government debt maturity. A large balance sheet means that the central bank's reserve supply is larger. Reserves are a liquid and safe investment for banks. They facilitate payments (deposit in/outflows) and serve as a safe interest-bearing store of value. Afonso, Duffie, Rigon and Shin (2022) show, for the US using data from 2010-2020, that even in recent years where reserve supply is high, banks rely on incoming payments to fund outgoing payments to conserve liquidity. They study payments in the Fedwire system which add up to around \$3T daily in recent years. Incoming payment over the prior 15 minutes is a strong predictor of outgoing payments over the next minute, more so the lower are reserve supply such as 2020. Reserves thus appear scarce in that they have value for liquidity management purposes even at high reserve supply. The evidence from Afonso et al (2022) imply that reserve demand is not saturated at any point during their sample.

Supplying plentiful liquidity is a common argument for a large central bank balance sheet. However, from the perspective of supplying liquid and safe assets, it is also relevant how the central bank supplies reserves. If reserves are supplied via central bank purchases of assets that are themselves liquid and safe, then the central bank's net liquidity/safety supply is reduced. Arrata, Nguyen, Rahmouni-Rousseau and Vari (2020) and more recently Schnabel (2023a) emphasize this issue in the euro area, with Eurosystem purchases contributing to a significant scarcity premium on some bonds, notably German bunds, purchased under ECB asset purchase programs. Hauser (2022) discusses the relevance of bond scarcity in the UK (especially for medium-term gilts). Similar issues are relevant in the US where the Federal Reserve over the longer run is planning to supply reserves exclusively via Treasury bond holdings. Krishnamurthy and Vissing-Jorgensen (2012) estimate the scarcity premium on Treasuries relative to Aaa-rated corporate bonds (driven by both high liquidity and extremely low default risk of Treasuries, as discussed below).

b) Interest rate volatility

A larger central bank balance sheet tends to be associated with lower short-rate volatility. Reserve demand is typically flatter at higher quantities, implying that a given reduction in reserve supply due to an increase in the autonomous factors will have a larger effect on the market rate-*IOR* spread at a lower initial reserve supply. The spike in short market rates on September 17, 2019, in the US happened after an increase in autonomous factors (the Treasury General Account increased due to debt issuance and tax receipts) which exacerbated the decline in reserves due to the Federal Reserve's QT. A larger supply of central bank reserves reduces the likelihood of such spikes and associated financial instability risks. Lopez-Salido and Vissing-Jorgensen (2023) estimate the level of reserves at which reserve supply would be as tight as in September 2019 given the current size of the banking sector as measured by deposits. They thus take an interest-volatility perspective to thinking about the implications of reserve demand for balance sheet reduction. They also

analyze take-up at the Federal Reserve's investment facility for non-banks (the ONRRP facility). As long as take-up at this facility is positive, fluctuations in the autonomous factors may only lead to fluctuations in the take-up at this facility, with no effect on the market interest rate.

c) Side effects of large central bank balance sheets

When a central bank increases reserves, banks need to fund their holding of these reserves. They can do so by lowering their non-reserve assets, or by increasing their liabilities. Central bank reserves may thus *crowd out* banking lending and may *crowd in* deposits or other liabilities. To the extent that banks are special and bank borrowers cannot easily replace bank funding with funding from non-banks, crowding-out of bank lending can lead to a welfare loss (for empirical evidence, see Diamond, Jiang and Ma (2022) and Chakraborty, Goldstein and MacKinley (2020)). As for liability crowd-in, more deposits may be beneficial if they provide liquidity/safety benefits to holders but to the extent the additional deposits are uninsured, they may lead to increased financial stability risks.¹¹ Financial stability risks may also increase if banks increase the risk of their non-reserve assets in response to larger reserve holdings.

d) Central bank profits and independence; headroom for future QE

A large central bank balance sheet exposes the central bank to duration risk if the assets held are of longer duration than the liabilities supplied (a large balance sheet may also expose the central bank to pre-payment risk and credit risk). As discussed above, under Ricardian equivalence (no convenience yields, non-distortionary taxation) this has no effect on welfare. With distortionary taxation, the maturity of the consolidated government debt matters, even aside from convenience yield effects. A large central bank balance sheet may be viewed as imposing too large a risk of a substantial tax increase or spending reduction, should interest rates increase. If sufficiently large, central bank losses may pose a threat to central bank independence. Aside from tax and spending implications, it may also be politically sensitive to pay large amounts of interest to banks (including foreign banks), especially when short market rates are below the interest rate on reserves.¹². Exante, these concerns imply that a large current balance sheet may be viewed as limiting the room for future QE if needed, as emphasized by Hauser (2022).

In my analysis, I will focus on (a), the effect of central bank balance sheet size and asset composition on the overall supply of liquid and safe assets. I do not model the effects of (b)-(d) on optimal balance sheet size. Relative to the convenience-maximizing balance sheet size that I calculate, concerns about interest rate volatility, (b), would tend to push toward a larger balance sheet, while concerns about central bank profit risk and QE headroom, (d), would result in a smaller balance sheet. For side-effects of QE on banks in terms of loan crowd-out and deposit crowd-in, the net

¹¹ Diamond, Jiang and Ma (2022) estimate that in US data covering QE1-QE3, each additional dollar of reserves led to 7 cents of additional deposits, 20 cents of reduction in bank loans (including mortgages), and 73 cents of reduction in banks' security holdings (net of wholesale funding and equity).

¹² In practice, central banks do appear to care about their profits. Goncharov, loannidou and Schmalz (2021) show that central banks are much more likely to report slightly positive than slightly negative profits.

effect on the desirability of a larger balance sheet is unclear. A central bank interested in (b)-(d) could use my convenience-maximization results as a starting point and increase/decrease reserve supply depending on policy preferences regarding (b)-(d). For any reserve supply under consideration, one can use my framework to calculate the convenience loss from deviating up/down from the convenience-maximizing reserve supply. This loss can then be weighed against any benefits from lowering interest rate volatility, or from preserving more room for future balance sheet expansion.

4

Framework: Convenience-maximizing reserve supply and asset choice

The Friedman rule for the optimal supply of central bank money (non-interest bearing cash) says that central banks should supply enough money to drive the convenience yield on money to zero. Since the convenience yield (on the last unit of money held) equals the foregone market interest rate, Friedman's prescription is thus to drive the market rate to zero. This is based on the underlying observation that producing money has almost no cost to the central bank. Welfare (the consumer's surplus plus the producer's surplus) is then simply the area under the money demand curve, which is maximized for r = 0.

Consider what this line of thinking implies about the optimal size of central bank balance sheets. The obvious answer is that the central bank should supply enough reserves that they are not scarce, meaning that the marginal value of reserves for banks in managing payments is zero. Accordingly, many central banks seek to assess the level of reserve supply at which reserves start to become scarce. This is often done via surveys of market participants' estimates of the "lowest comfortable level of reserve balances" (LCLoR), also known as "floor required excess liquidity" (FREL), or the "Preferred Minimum Range of Reserves" (PMRR) see, e.g., Keating et al (2019) for the US, Aaberg et al (2021) for the euro area, and Bank of England (2023) for the UK.

However, this argument ignores central bank asset holdings. If these assets have a convenience yield of their own, the central bank's cost of producing reserves is not zero. After giving a bit of background on bond convenience yields, this section lays out the core result for how the central bank should trade off convenience yields on reserves and asset holdings. The result accounts for bank balance sheet costs and bank funding of reserves. After stating the result, I argue that it is valid regardless of the exact funding mix banks use to fund reserves.

4.1 Background on bond convenience yields

By way of background, by "convenience yield" I mean security benefits over-andabove any interest and principal payments. Krishnamurthy and Vissing-Jorgensen (2012) (KVJ) lay out a framework for convenience yields on bonds in which they can arise due to investors placing a special value on the high liquidity of an asset or on its very low default risk (safety). Bond liquidity results in saved *transactions costs*. This is similar to the convenience yield banks obtain from the liquidity of reserves. Very high safety results in saved *information costs*, as investors do not need to do credit risk analysis. Convenience yields from liquidity and safety imply that investors are willing to invest in the asset even at a lower yield from interest and principal payments.

I illustrate convenience yields on bonds in Figure 2, which plots the relation between bond yields and default risk. Convenience benefits on very low-risk assets are illustrated by the curved part of the relation. Some investors appear willing to accept a yield on very safe assets that is below the yield implied by their low risk and the "normal" yield-risk relation based on higher-risk assets, illustrated with the solid straight line. These very safe assets also tend to be very liquid.

Figure 2

Convenience yields on bonds -- illustration for Treasuries



Note: For simplicity. I illustrate the relation between yield and default probability absent convenience effects as a straight line. In practice it is slightly convex: Thinking of a 1-period zero-coupon bond with face value F, yield y, default probability p, and zero recovery in default, the price is P=F/(1+y)=(1-p)*F/(1+E(r)). This implies 1+y=(1+E(r))/(1-p) which means that y is approximately, but not exactly, equal to E(r)+p.

Using a Treasury bond as an example of a very low default risk asset, the convenience yield is the vertical distance between the red dots on the y-axis. The yield spread between the yield on a corporate bond without a convenience yield and the yield on the Treasury bond is then determined by the default component of the corporate bond yield (which increases the corporate bond yield) and the convenience yield on the Treasury bond (which lowers the Treasury yield)

 $y^{Inconv.corp.bond} - y^{Treasury} = \underbrace{v'_T(T)}_{\text{Treasury}}_{\text{conv.yield}} + \underbrace{\text{Default component}}_{\text{Spread for large}}_{\text{Treasury supply}}$

The Treasury convenience yield declines with Treasury supply. In Figure 2, with fully saturated Treasury convenience demand, the curved and dashed lines would overlap. The Treasury bond would plot at the top red dot and the yield spread relative to corporate bonds without convenience yields would be driven only by the

default component of the corporate bond yield. KVJ exploits this idea to decompose the yield spread between Aaa-rated corporate bonds and Treasuries into the Treasury convenience yield and the default risk component (this component also captures effects of differential tax treatment on the two types of bonds). They regress the Aaa-Treasury yield spread on Treasury Debt/GDP, allowing for a horizontal asymptote. The estimated horizontal asymptote (i.e., the value of the spread when Treasury debt is plentiful, and safety and liquidity demand satiated) is a measure of the default component of the spread. The remaining spread then measures of the convenience yield on Treasuries. They estimate an average Treasury convenience yield relative to Aaa-rated corporate bonds of 46 bps over the period 1919-2008. Aaa-rated corporate bonds may themselves have some appeal to safety investors, implying that Aaa bonds may plot on the curved line a bit to the left of the "Inconvenient corporate bond" point. Using the spread between Baa-rated corporate bonds and Treasuries, adjusted for the default component on the Baa bonds, KVJ find a larger Treasury convenience yield averaging 73 bps over the same period.

In terms of the economics underlying convenience yields from safety, the issue of saved information costs from not having to do credit risk analysis is related to the literature on limited participation (e.g., Vissing-Jorgensen (2003)). Assets with low default risk also serve as particularly good collateral for borrowing. Distinguishing between short and long bonds, KVJ furthermore emphasize that for long-maturity bonds, investors like pension funds (and some insurance companies) may have a special demand for low-default risk long-term payoffs to back long-term nominal liabilities. This is a type of preferred-habitat demand, focused on low-default risk assets. Whether convenience yields are due to liquidity effects or safety effects will not matter for my arguments below.

4.2 Trading off convenience yields on reserves and asset holdings

Suppose a central bank (cb) supplies reserves *R* via bond holdings B^{cb} . Denote the total convenience value from reserves by $v_R(R)$ and the total convenience value from bonds by $v_B(B^{priv})$, where B^{priv} is the private sector's bond holdings and $B^{cb} + B^{priv} = B$, the total bond supply. For now, ignore the fact that $v_R()$ and $v_B()$ have additional arguments because convenient assets are more valuable when the banking sector or the economy is larger. I return to these issues below.¹³

The central bank's balance sheet must balance, so $B^{cb} = R + A$, with A denoting the autonomous factors on the central bank balance sheet:

Assets	Liabilities
Bonds, B ^{cb}	Reserves, R
	Autonomous factors (cash, government deposits, etc.), A

³ Also, notice that I use separate convenience values for reserves and bonds rather than an aggregate. For the US, empirical proxies for the convenience yield on reserves and government bonds or bills are slightly negative correlated over my sample period from 2009M1-2023M4, as I will discuss below, suggesting that the assets do not enter a common aggregate.

Total convenience available to the private sector via its holdings of reserves and bonds is then

$$v_R(R) + v_B(B^{priv}) = v_R(R) + v_B(B - B^{cb}) = v_R(R) + v_B(B - R - A).$$

Should a convenience value for the central bank be added above? In other words, do citizens benefit if their central bank holds assets that are particularly liquid or safe? In the case of a central bank holding domestic bonds to maturity, the central bank (and thus its ultimate owners, the citizens) does not benefit from holding liquid assets.¹⁴ Similarly, citizens may not benefit if their central bank holds very safe assets as the central bank would likely to able to understand and invest in other types of assets without large associated costs. I will thus proceed to assume that total convenience of citizens is as stated above.

As discussed in the introduction, citizens may have reasons to prefer that their central bank hold one type of assets over another. They may have a strong preference for their central bank not interfering with the allocation of credit among private borrowers by buying anything other than government bonds, a case that may fit the Federal Reserve. One could think of this as citizens getting a disutility from the Federal Reserve holding non-Treasuries. Alternatively, citizens may have a strong preference for their central bank not funding the government (or the governments of other countries), perhaps better describing the euro area. Whatever preferences citizens may have over their central bank's asset holdings, I will model these as a constraint on the central bank's asset choice and analyze how to set reserve supply to maximize convenience given that constraint.

For most of the post-GFC period, the effective federal funds rate and a host of other short-term rates in the US have been below the interest rate on reserves. From equation (1) this implies that φ must be substantial. If not, competition between banks for raising funds from investors without access to interest-bearing reserves would push up short-term rates to equal the interest rate on reserves plus $v'_R(R)$ which is non-negative. While balance sheet costs are thus clearly relevant for banks, how do we think of these costs from a welfare perspective? Is φ a social cost?

From banks' perspective, the balance sheet cost is typically thought to emerge from capital requirements such as the Supplementary Leverage Ratio that requires banks to use a minimum fraction of equity financing when expanding their balance sheets. Equity financing is more expensive for banks because equity, unlike short-term bank debt, does not appeal to safety or liquidity investors. Capital requirements are, in turn, imposed on banks because there is an externality from banks funding themselves with short-term debt.¹⁵ Stepping back and thinking about how regulators set capital requirements, if they are set optimally, the banks' perceived balance sheet

¹⁴ This contrasts with a foreign central bank holding these bonds for potential FX interventions.

¹⁵ Stein (2012) explains how this externality can emerge from banks not internalizing the effect of their fire-sales of assets (in response to short-term debt rollover problems) on the value of other banks' assets. This results in too much short-debt debt issuance, too large fire-sale discounts, and a social welfare loss because patient investors who buy in a fire-sale could alternatively invest in new investment projects with positive welfare. The higher the fire-sale discount, the higher the hurdle rate for such investments, and the worse the underinvestment.

cost (the cost of using more equity financing than they would prefer) is set to equal social cost of larger bank balance sheets (from the short-term debt externality).

There is a related debate about whether capital relief should be given for reserve holdings given their safe and liquid nature. Both the Federal Reserve and the ECB gave SLR relief for reserves during the COVID crisis but have since ended such relief.¹⁶ Implicitly, this suggests that regulators perceive a social cost of larger bank balance sheets even for reserves. An argument supporting this view is that while a narrow bank that funded reserve holdings with short-term debt would always be able to cover funding outflows with reserves, aggregate reserves must be held by someone. Therefore, an aggregate funding outflow from the overall banking sector (to something other than another central bank liability, notably cash) necessitates either sales of non-reserve assets by banks or by the central bank. This makes SLR relief less attractive from a macro-prudential than a micro-prudential perspective.

Given these arguments, I proceed to assume that φ represents a social cost and therefore should be subtracted when maximizing convenience supply net of balance sheet costs. Welfare from convenient assets, net of balance sheet costs, is then

$$[v_R(R) - \varphi R] + v_R(B^{priv})$$

$$= [v_R(R) - \varphi R] + v_B(B - B^{cb}) = [v_R(R) - \varphi R] + v_B(B - R - A).$$

If the central bank (like the ECB) can supply reserves via holding assets *without* convenience value, the central bank's convenience maximization problem is then

$$\max_{R} v_{R}(R) - \varphi R.$$

If the central bank (like the Federal Reserve's announced longer-run plans) holds assets *with* convenience value to the private sector, the central bank's convenience maximization problem is

$$\max_{R} v_{R}(R) - \varphi R + v_{B}(B - R - A)$$

Taking first-order conditions, we have the following result.

Result 1 (Convenience-maximizing reserve supply).

(*A*) If a central bank holds assets without convenience yields, then the conveniencemaximizing supply of central bank reserves, R^A, solves

$$\mathbf{v}_{\mathbf{R}}'(\mathbf{R}) - \varphi = \mathbf{0}.$$

(B) If the central bank holds bonds with convenience yields (to market participants), then the convenience-maximizing supply of central bank reserves, R^B , solves

$$v_R'(R) - \varphi = v_B'(B - R - A).$$

¹⁶ By contrast, the Bank of England gives capital relief for reserves.

 R^A depends on the shape (and shifters) of the convenience yield function $v'_R(.)$. R^B also depends on the shape (and shifters) of the convenience yield function, $v'_B(.)$, as well as the aggregate supply of bonds *B* and the autonomous factors *A*.

Figure 3 illustrates the moving parts of Result 1(A). In Panel a left, the total convenience value of reserves is the area between the reserve demand curve and the *IOR*. It has two components. The "consumers' surplus" area (CS) indicates how much more banks value reserves than the amount r^R it costs them to borrow R at the market rate r to invest in reserves. For a given unit of reserves, the consumers' surplus is $[IOR + v'_R(R) - \varphi - r]$. The "producers' surplus" area (PS) indicates the central bank's benefit from funding reserves at a rate *IOR* which is below the short-term market rate r (in the example graphed). For a given unit of reserves, the producers' surplus is [r - IOR]. The sum of the consumers' surplus and the producers' surplus is the total convenience value of reserves (net of φ), which is $[IOR + v'_R(R) - \varphi - r] + [r - IOR] = v'_R(R) - \varphi$ for a given unit of reserves. Figure 3, Panel a right, changes the y-axis to subtract the IOR in order to focus on the consumers' and producers' surpluses is then simply the area between the demand curve and the horizontal axis. It is clearly not maximized for the reserve supply in Figure 3, Panel a.

Figure 3







b) Convenience-maximizing reserve supply in Result 1(A)



Note: In panel a), "CS" refers to the consumers' surplus and "PS" to the producers' surplus. In panel b), R^A maximizes the convenience from reserves (CS+PS) by setting $v'_R(R) - \varphi = 0$.

Figure 3, Panel b, shows the convenience-maximizing reserve supply from Result 1(A). This supply maximizes the sum of the consumers' and producers' surpluses from reserves by setting $v'_R(R) - \varphi = 0$ and thus $v'_R(R) = \varphi$. Point *A* illustrates this

outcome. Even in this case (Case A) where the central bank can supply reserves by holding only assets without convenience yields, it is not the case that fully saturating reserve demand to set $v'_R(R) = 0$ is optimal. This would result in a too high reserve supply in the sense that the convenience yield net of balance sheet costs was negative for the units of reserves in excess of R^A .

Figure 4 illustrates Result 1(B). Panel a right illustrates the convenience yield on bonds using Treasuries T as an example (the same graph is relevant for Bunds or other very safe euro area bonds). The y-axis in this figure is the convenience yield on a given unit of Treasuries held by the private sector, $v'_{T}(T^{priv})$. It equals the spread between the yield on inconvenient bonds with similar maturity as Treasury bonds and the yield on the Treasury bonds, with this spread adjusted for the default component due to corporate default risk. The total convenience value on Treasuries is the area between the demand curve and the horizontal axis, which again can be decomposed into the consumers' and producers' surplus. I illustrate in Figure 4, Panel a right, the point A that corresponds to a reserve supply of R^A. For the case illustrated, R^A results in central bank holdings of Treasuries that are so large as to imply a substantial scarcity of Treasuries for the private sector as measured by $v'_T(T^{priv})$. Since $v'_R(R) - \varphi = 0$ and $v'_T(T^{priv}) > 0$ at a reserve supply of \mathbb{R}^A , a central bank that can only supply reserves via Treasuries would maximize overall convenience from reserves and Treasuries by lowering reserve supply, thereby freeing up more Treasuries for the private sector to hold. This is illustrated with the points labelled B in both the left and right figure, corresponding to Result 1(B). R^{B} equates $v'_R(R) - \varphi$ and $v'_T(T^{priv})$. This maximizes the sum of the consumers' and producers' surpluses from reserves and Treasuries combined.

The key insight of Result 1(B) is that the supply curves in the left and right figures of Figure 4, Panel a, are linked when reserves are supplied via central bank holdings of convenient bonds (Treasuries in the example graphed). A decrease in reserves moves the reserve supply curve S to the *left* and increases the convenience yield on reserves. At the same time, the decrease in reserves moves the Treasury supply curve (for holdings available to the private sector) to the *right* and decreases the convenience yield on Treasuries. Figure 4, Panel b, further illustrates Result 1(B), graphing both reserve and Treasury convenience yields as functions of reserve supply.

It should be noted that $v_T(T^{priv})$ (and $v_B(B^{priv})$ in general) refers to the convenience from bond holdings of anyone other than the central bank. This will include foreigners (private and official). Similarly, $v_R(R)$ includes reserve holdings by US branches of foreign banks which may be foreign owned. Implicitly I do welfare optimization assuming foreigners matter equally. If one wanted to calculate convenience-maximizing reserve supply focusing only on the convenience benefits obtained by domestic citizens, one would need to model only holdings by domestic residents/domestically owned banks, $v_{B^{Dom.held}}(B^{Dom.held})$ and $v_{R^{Dom.held}}(R^{Dom.held})$. The first-order condition in Result 1(B) would change to

$$v'_{R^{Dom,held}}(R - R^{Foreign \ owned}) * \left(1 - \frac{dR^{Foreign \ owned}}{dR}\right)$$
$$= v'_{B^{Dom,held}}(B - B^{Foreign \ owned} - R - A) * \left(1 + \frac{dB^{Foreign \ owned}}{dR}\right)$$

I will not pursue this further here. It is likely that countries value the benefits of citizens of other countries to some extent (perhaps fully in that they may expect to get something in return, such as in the US case the fact that these benefits help support the status of the dollar).

Figure 4

Convenience-maximizing reserve supply: Illustrating Result 1(B)





b) Graphing both convenience yields against reserves



Note: In panel a), R^{B} maximizes total convenience from reserves and Treasuries by setting $v_{R}'(R) - \varphi = v_{T}'(T^{priv})$.

Comparative statics

Figure 5 provides comparative statics. I illustrate which variables shift the convenience demand schedules and thus affect the convenience-maximizing reserve supply.

Deposits, $\frac{dR^A}{dD} > 0$ and $\frac{dR^B}{dD} > 0$: An increase in bank deposits shifts $v'_R(R) - \varphi$ up as higher deposits increase the expected marginal value of reserves for managing deposit outflows with low transactions costs. Figure 5, Panel a left, shows how a deposit increase leads to an increase in the convenience-maximizing reserve supply in Result 1(A). Figure 5, Panel a right, shows how a deposit increase affects the convenience-maximizing reserve supply in Result 1(B). The convenience-maximizing reserve supply increase is smaller in Result 1(B) than in Result 1(A) because of the

positive slope of the $v'_B(B - R - A)$ (with Treasuries *T* again used as the example of *B* in Figure 5). Increasing reserves in response to higher deposits is less attractive when reserves are increased by buying bonds with a convenience yield.

Figure 5

Comparative statics for convenience-maximizing reserve supply



b) Effects of GDP in Result 1(B)



c) Effects of bond supply and autonomous factors in Result 1(B)



GDP, $\frac{dR^B}{dGDP} < 0$: Figure 5, Panel b, illustrates the effect of higher GDP in Result 1(B). When the economy is larger, convenient bonds are scarcer at each level of reserves, so the $v'_B(B - R - A)$ function shifts up. This reduces the convenience-maximizing reserve supply in Result 1(B) which moves from point 1 to point 2.

Convenient bond supply, B, $\frac{dR^B}{dB} > 0$: Figure 5, Panel c, shows the effect of a higher convenient bond supply on the convenience-maximizing reserve supply in

Result 1(B). Starting from point 1, an increase in convenient bond supply decreases the bond convenience yield at each level of reserves and thus shifts down $v'_B(B - R - A)$. At the initial reserve level, the net convenience yield on reserves now exceeds the bond convenience yield. Equating the net convenience yield on reserves and the convenience yield on bonds requires more reserves as this will increase $v'_B(B - R - A)$ (along its new curve) and decrease $v'_R(R) - \varphi$, shifting the equilibrium to point 2. Result 1(B) can be used to derive the magnitude of the convenience-maximizing reserve-response to increased convenient bond supply. Changing *R* and *B* such that $v'_R(R) - \varphi = v'_B(B - R - A)$ still holds, the reserveresponse will depend on how sensitive each of the convenience yields are to changes in their arguments:

$$\frac{dR^B}{dB} = \frac{v_B''(B - R - A)}{v_R''(R) + v_B''(B - R - A)} \le 1$$

Both $v''_B(B - R - A)$ and $v''_R(R)$ are non-positive. If $v''_R(R)$ is large in absolute value, the convenience yield on reserves declines a lot with additional reserve supply, and the convenience-maximizing reserve increase in response to additional convenient bond supply is more muted. If $v''_B(B - R - A)$ is large in absolute value, the convenience yield on bonds declines a lot with additional convenient bond supply, and the convenience-maximizing reserve increase in response to additional convenient convenience yield on bonds declines a lot with additional convenient bond supply, and the convenience-maximizing reserve increase in response to additional convenient convenient bond supply is larger.

Autonomous factors, $\frac{dR^B}{dA} < 0$: The effect of an increase in autonomous factors in Result 1(B) is also illustrated in Figure 5, Panel c. In contrast to the effect of a higher supply of convenient bonds, an increase in the autonomous factors increases the bond convenience yield for each level of reserves (because the central bank now holds more convenient bonds, leaving fewer for the private sector). With higher autonomous factors *A*, the net convenience yield from reserves is below the bond convenience yield at the initial reserve level. In response, a convenience-maximizing central bank should decrease reserves (and thus its convenient bond holdings), as illustrated with point 3.

Two maturities of convenient bonds

A central bank that supplies reserves via purchases of convenient bonds has access to a range of bond maturities. Consider a case with two bond maturities that each have their own convenience function because they differ in maturity and some investors have preferred habitat demand for very safe long bonds (bond 1) while others have preferred habitat demand for very safe short bonds (bond 2). Using B_1^{priv} and B_2^{priv} to denote the private sector's holdings of each type of convenient bonds, this could be captured as follows:

$$\begin{aligned} & \max_{R,B_1^{cb},B_2^{cb}} v_R(R) - \varphi R + v_{B_1}(B_1^{priv}) + v_{B_2}(B_2^{priv}) \\ & \text{st. } B_1^{priv} = B_1 - B_1^{cb} \\ & B_2^{priv} = B_2 - B_2^{cb} \\ & B_1^{cb} + B_2^{cb} = R + A \end{aligned}$$

Substituting in the constraints we get:

$$\max_{B_1^{cb}, B_2^{cb}} v_R(B_1^{cb} + B_2^{cb} - A) - \varphi[B_1^{cb} + B_2^{cb} - A] + v_{B_1}(B_1 - B_1^{cb}) + v_{B_2}(B_2 - B_2^{cb})$$

The first-order conditions are:

$$v_{R}'(B_{1}^{cb} + B_{2}^{cb} - A) - \varphi = v_{B_{1}}'(B_{1} - B_{1}^{cb})$$
$$v_{R}'(B_{1}^{cb} + B_{2}^{cb} - A) - \varphi = v_{B_{2}}'(B_{2} - B_{2}^{cb})$$

Convenience-maximization thus implies equalizing the net convenience yield on reserves and the convenience yields on bond 1 and bond 2. For example, if the Federal Reserve supplied reserves with purchases of Treasury bills and Treasury bonds, and it observed that Treasury bills had a lower convenience yield than Treasury bonds, then it could increase the supply of convenience to the private sector by reallocating some of its portfolio from bonds towards bills. Similarly, if the Federal Reserve observed a higher convenience yield on both bills and bonds than the net convenience yield on reserves, it could increase convenience by reducing its reserve supply.

One may wonder whether reserves and bills should enter the same convenience measure, e.g., $v_{short}(R + kB_1^{cb})$ for some constant k. This may be appropriate if reserves and bills were both held by the same investor type. However, d'Avernas and Vandeweyer (2023) argue theoretically and empirically that for high reserve supply and modest bill supply, banks tend to not hold bills which are instead held by investors without access to reserves (shadow banks such as money market funds). The pricing of reserves then disconnects from the pricing of bills, with the spread between reserves and inconvenient assets driven by banks' convenience yield from reserves and the spread between bills and inconvenient assets driven by shadow banks' convenience yield from bills.

The prescription of this sub-section emphasizes the need to think about not only liquidity but also safety effects. Since Treasury bills tend to be more liquid than Treasury notes and bonds, maximizing the overall liquidity services to the private sector would suggest supplying reserves by buying notes or bonds. However, bonds and notes have large convenience yields due to safety effects and buying them may be more detrimental to overall convenience supply to the private sector than buying bills! Optimally, the central bank (and the fiscal authority) maximizes convenience supply by equalizing convenience yields on all securities supplied, i.e., reserves, bills and notes/bonds.

What if the ECB supplies reserves via a mix of inconvenient and convenient assets?

I have emphasized that historically (pre-sovereign debt crisis and QE) the ECB supplied reserves via loans to banks. This thus appears to be a feasible option going forward. Such loans are unlikely to appeal to safety or liquidity investors. The ECB's convenience-maximization problem then maps to Result 1(A). How would the ECB's convenience-maximization problem change if, for reasons outside my framework, the

ECB decided to supply reserves with a mix of bank lending and government bond purchases?

Suppose the ECB's assets were a pre-set mix of government bonds and loans to banks with weights ω and 1- ω and that bank loans carry no convenience yields (to the ECB or others). Assume that the ECB's government bond holdings were a preset mix of the *N* euro area countries' government bonds. Denote portfolio weights within the government bond portfolio by $\alpha_1, ..., \alpha_N$, where $\sum_{i=1}^N \alpha_i = 1$. Suppose that a subset *M* of the *N* countries' government bonds had a convenience yield, and the rest did not. Assume a convenience yield function $v_B (B_1^{priv} + k_2 B_2^{priv} + \cdots + k_M B_M^{priv})$, where $M \leq N$. The first-order condition for convenience-maximization would be

$$v_{R}'(R) - \varphi = v_{B}'(B_{1}^{priv} + k_{2}B_{2}^{priv} + \dots + k_{M}B_{M}^{priv}) * \omega * (\alpha_{1} + k_{2}\alpha_{2} \dots + k_{M}\alpha_{M})$$

For example, if bond 1 was German bunds, and these were the only ones with a convenience yield, the first-order condition for convenience-maximization would set the convenience yield on reserves equal to the convenience yield on bunds times $\omega \alpha_1$

 $\nu'_R(R) - \varphi = \nu'_B(B_1^{priv}) * \omega * \alpha_1.$

4.3 Result 1 holds regardless of banks' funding mix for reserves

In deriving Result 1, I did not discuss how banks fund their reserve holdings. This subsection argues that Result 1 holds regardless of the specific mix of funding used by banks to fund reserves.

Bank can fund reserves by (a) raising more liabilities, (b) reducing lending (or selling other assets than those purchased by the central bank), or (c) selling assets to the central bank when it buys assets to supply reserves. At the welfare maximum, banks use each approach until they are equally costly. My Result 1 implicitly focuses on banks' use of (a) since I subtract φ from $v'_R(.)$. The fact that banks also use (b) and (c) does not change the fact that the first-order condition from (a) that I rely on should hold.

To see the moving parts of the argument, start from the overall welfare generated by reserves, bonds, loans/bank security holdings, and bank deposits:

$$v_R(R,D) + v_{B,banks}(B_{banks}, .) + v_{B,nonbanks}(B_{nonbanks}, .) - \varphi * (R + L + B_{banks}) + \lambda(L)$$

where *R* is reserves, *D* is deposits, B_{banks} and $B_{nonbanks}$ are holdings of banks and non-banks (including households) of convenient bonds, *L* is loans (including bank holdings of other types of securities). $R + L + B_{banks}$ equals total bank assets which equal total bank liabilities. $\Lambda(L)$ denotes the welfare (producers and consumers surplus) from bank loans (above and beyond the balance sheet cost). I note that there is not a separate term for deposits in the welfare expression since φ is the social cost of bank liabilities net of any benefits households may get from holding the deposit part of these liabilities. Consider now each of the three funding approaches and focus on Result 1(B) in which the central bank supplies reserves by holding bonds with convenience yields. In case (a) (more bank liabilities), in response to higher reserves, banks' reserve holdings increase, banks liabilities increase, and convenient bond holdings of non-banks decrease. The first-order condition for welfare maximization is:

$$v'_{R}(R,D) - v'_{B,nonbanks}(B_{nonbanks}, .) - \varphi = 0$$
(3)

where $v'_R(R,D)$ is the derivative with respect to *R* for given *D*. Non-bank balance sheet size is unchanged overall, with lower holdings of convenient bonds, but higher holdings of bank liabilities.

In case (b) (reduced bank lending/securities holdings), the effect of higher reserves is to increase banks' reserve holdings, decrease banks' lending, and decrease convenient bond holdings of non-banks. The first-order condition for welfare maximization is:

$$v'_{R}(R,D) - v'_{B,nonbanks}(B_{nonbanks}, .) - \lambda'(L) = 0$$
(4)

Non-bank balance sheet size is unchanged overall, with lower holdings of convenient bonds, but higher holdings of other assets, purchased from banks. Importantly, $\lambda'(L) = \varphi$, so (4) becomes the same as (3). To see this, observe that in the competitive equilibrium (which maximizes welfare by the first welfare theorem), households' FOC for borrowing is

where r^{Loan} denotes the interest rate on bank loans. Banks' FOC for lending is:

 $r^{Loan} = marginal \ cost + \varphi$

Therefore, $\lambda'(L)(= marginal \ benefit - marginal \ cost) = \varphi$.

In case (c) (banks sell convenient bonds to the central bank), higher reserves imply that banks' reserve holdings increase while banks' convenient bond holdings decrease. Banks incur no balance sheet costs from additional reserves since their assets and liabilities are unchanged overall. The first-order condition for welfare maximization is:

$$v'_{R}(R,D) - v'_{B,banks}(B_{banks},.) = 0$$
(5)

For the last unit held, banks and non-banks must value convenient bonds the same. Since banks incur balance sheet costs on all their assets, banks' bond holdings must at the welfare-optimum be sufficiently small that the bond convenience yield of banks exceeds that of non-banks by φ . Therefore, (5) becomes (3) since

 $v'_{B,banks}(B_{banks},.) = v'_{B,nonbanks}(B_{nonbanks},.) + \varphi.$

Because (3), (4) and (5) all hold at the optimum (as long as each funding approach is used to some extent), we can assess whether welfare is maximized by measuring whether $v'_{R}(R) - \varphi = 0$ (Result 1(A)) or $v'_{R}(R) - \varphi = v'_{B}(B - R - A)$ (Result 1(B)),

regardless of the exact mix with which banks fund reserves. That does not mean that it is irrelevant from a welfare perspective how reserves are financed, but that we can assess welfare-maximization based on the same conditions regardless. From a welfare perspective, if reserves crowd out lending or holdings of securities this affects welfare negatively. If reserves crowd in bank liabilities this may increase welfare to the extent that some of these liabilities have safety and liquidity benefits to their holders (notably deposits), but it may decrease welfare to the extent that higher bank liabilities then crowd out other investments that buyers of bank liabilities could hold instead.

Mapping Result 1 to data, I will measure $v'_{B,nonbanks}(B_{nonbanks}, .)$ using a spread between inconvenient bonds and convenient bonds that are both held by non-banks.¹⁷ $v'_{R}(R) - \varphi$ will be measured using a spread between reserves and an inconvenient security that is a liability for banks (federal funds since banks post-GFC borrow from government sponsored enterprises in this market).

5

Estimating the convenience-maximizing reserve supply for the US

This section estimates the convenience-maximizing reserve supply for the US as of April 2023 based on both Result 1(A) (reserves supplied via inconvenient asset holdings) and Result 1(B) (reserves supplied via convenient asset holdings). The Federal Reserve plans to supply reserves via Treasury holdings over the longer run so the empirical estimate of convenience-maximizing reserve supply based on Result 1(B) is the more relevant one. This estimate is much smaller than the estimate based on Result 1(A). My estimates thus imply that the convenience-maximizing balance sheet size of the Federal Reserve is much smaller than if the Federal Reserve was able (politically) to supply reserves via inconvenient asset holdings. The next section turns to estimates for the ECB for which supply of reserves via inconvenient asset holdings appears preferable from a political perspective.

5.1 Yield spreads

I illustrate the yield inputs to my analysis for the US in Chart 1 which is based on monthly average data. In the top left figure, the red line shows the effective federal funds rate] (EFFR) – IOR spread. This spread is negative over most of the post-GFC sample, except around September 2019 which includes the infamous September 17,

¹⁷ A recent literature including Klingler and Sundaresan (2019) and Du, Hebert and Li (2022) compares Treasury yields to yields on interest rate swaps and finds that long-maturity Treasury yields are above swap rates in the post-GFC period. This is reconciled by banks (or dealer banks) being short swaps and long Treasuries, with a spread opening up to cover balance sheet costs. This is different from measurement of v'_{B,nonbanks}(B_{nonbanks}.) in my setting which measures whether bonds purchased by the central bank have convenience yields relative to other assets held by nonbanks, not relative to derivatives.

2019, spike in repo rates and in the effective federal funds rate. The value of the EFFR-IOR spread for April 2023 is -7 bps.

Chart 1

US: Yield spreads on reserves and Treasuries (relative to inconvenient assets)





Combined chart

1.25 1.5 1.75

.75 'n

25

Spreads on Treasuries relative to corporate securities adjusted for corporate CDS (and callability)



Note: All data in the top left, top right, and bottom left graphs are from FRED. Commercial paper is AA-rated, financial. In the bottom right graph, OAS and CDS data are from Bloomberg. Both refer to investment-grade corporate bonds. OAS data are for the ICE BofA US Corporate Index (which is investment grade). CDS data are for Markit's investment-grade index (MARKIT CDX.NA.IG).

The EFFR-IOR spread is a measure of $v'_R(.) - \varphi$.¹⁸ We can infer that the balance sheet cost φ must be substantial and at least as large in absolute value as the most negative value of the EFFR-IOR spread. This lowest value is around -20 bps. While the balance sheet cost may vary somewhat over time, to get a sense of the level of $v'_{R}(.)$, I illustrate EFFR-IOR+20 bps in the top left figure in orange. The line suggests that reserves had a scarcity value of around 25 bps in September 2019, making it less puzzling that yields spiked in the sense that reserves were less plentiful than the EFFR-IOR spread may suggest.

Chart 1, top right, graphs the yield spread between corporate securities and Treasuries, using long-maturity bonds (dark blue line) or 3-month maturity securities

¹⁸ I do not attempt to estimate any default component of EFFR. It is likely modest since the series used is the median volume-weighted rate. If there is default risk in EFFR, then EFFR-IOR over-estimates $v'_{R}(.) - \varphi$. On the other hand, federal funds may be safe enough to have some appeal to safety investors in which case EFFR-IOR under-estimates $v_R'(.) - \varphi$. It is unclear which of these effects dominates.

(light blue line). The long-maturity spread is based on Aaa-rated corporate bonds with 20 or more years to maturity and the Treasury yield is the 20-year yield. This spread is large across the sample, much larger than the short maturity spread which is positive but modest over most of the sample. Some of the Aaa-Treasury (and Commercial paper – T-Bill) spread reflects default risk in the corporate securities. I will account for this below by estimating the default component as the asymptote of the Aaa-Treasury spread for large Treasury supply (and thus saturated Treasury demand). The Aaa-Treasury spread is 66 bps in April 2023. Below, I estimate an asymptote of about 31 bps, thus implying a Treasury convenience yield from this substantially larger than the currently negative convenience yield on reserves, net of the balance sheet cost.

Chart 1, bottom left, graphs the EFFR – IOR spread along with the two corporate-Treasury spreads. The EFFR – IOR spread has correlation of 0.03 with the Commercial paper – T-bill spread and a correlation of -0.31 with the Aaa corporate – Treasury spread, consistent with my use of separate convenience value functions (v) for reserves and bills/bonds.

As an alternative approach to measuring the Treasury convenience yield, one can use the fact that measures of the credit risk in corporate bonds are available in recent years from credit default swap markets. The Treasury convenience yield can then be measured as follows:¹⁹

(Corporate yield - Treasury yield) - (Credit default swap rate for corporate bonds).

This approach is due to Longstaff, Mithal and Neis (2005) and assumes that a spread position plus CDS protection is a package which is neither liquid nor appeals to safety investors. In recent years one can also adjust the corporate-Treasury yield spread for the fact that some corporate bonds may be callable. This is done by using option-adjusted spreads (OAS) of corporates relative to Treasuries, instead of the raw spread (Corporate yield - Treasury yield). I pursue this alternative approach in Chart 1, bottom right, using OAS and CDS data for investment-grade corporate bonds. The OAS-CDS spread can be constructed for bonds of various maturities. For the OAS on bonds of 3 to 5-year maturity, I subtract the average of the 3-year and 5-year CDS rates. Similarly, for the OAS of bonds of 5 to 7-year maturity (subtracting the average of the 5-year and 7-year CDS rates) and for the OAS of bonds of 7 to 10-year maturity (subtracting the average of the 7-year and 10-year CDS rates). The three OAS-CDS spreads are between 51 and 62 bps in April 2023 and generally similar across maturities. This suggests that Treasuries have a convenience yield not only for long maturities but also down to at least the 3-year maturity.²⁰ The slightly lower Treasury convenience yield when using Aaa-rated corporate bonds (35 bps in April 2023) than all investment-grade corporate bonds is

¹⁹ I do not adjust for the CDS-rate on Treasuries. While it is not zero, it is generally thought to mainly reflect the risk of delayed payments due to debt ceiling negotiations, as opposed to outright default.

²⁰ The weighted average maturity of marketable Treasury debt outstanding is 74 months as of 3/31/2023, according to the US Treasury.

consistent with Aaa-rated corporate bonds themselves having some convenience yield.

Below, I will focus on the long-maturity Aaa-Treasury spread (adjusted for the asymptote). I use this spread is a stand-in for the convenience yield of Treasuries in general. This is clearly an approximation. Even if notes and bonds have fairly similar convenience yields (as suggested by the apparent lack of term structure of the convenience yield from the OAS-CDS spreads), the Commercial paper – T-bill spread is lower than the Aaa-Treasury spread. One could consider using a weighted average of the Aaa-Treasury spread and the Commercial paper- T-bill spread. On the other hand, one could also consider using corporate bonds rated lower than Aaa given that the Aaa-Treasury spread may understate the Treasury convenience yield. Since these two adjustments would go in opposite directions, for simplicity, I will proceed with simply using the Aaa-Treasury spread as a proxy for the convenience yield of Treasuries overall.

A related issue is that my analysis implicitly assumes the Federal Reserve holds a typical Treasury portfolio of bonds, notes, and bills. Given the lower convenience yield on bills, the Federal Reserve could likely increase its overall convenience supply by shifting some of its assets from a typical bundle of all Treasuries to a bundle that was over-weight in Treasury bills. As noted in section 4.2, for given overall government debt, convenience supply is maximized when convenience yields on bills, bonds/notes and reserves are equalized. I do not explore that here. As the Federal Reserve allocated more of its assets to bills, the convenience yield on bills would increase from currently close to zero but it is difficult to assess by how much. I leave it to future work to estimate the current Treasury bill demand function to assess this intermediate case. A large Federal Reserve portfolio of bills raises separate concerns about being able to roll over the portfolio given the fluctuations in bill supply.²¹

To estimate convenience-maximizing reserve supply, we need estimates of the functional forms of $v'_R(.) - \varphi$ and $v'_T(.)$. I turn to this next based on two earlier papers of mine, updated with the latest available data.

5.2 Estimating reserve demand

Lopez-Salido and Vissing-Jorgensen (2023) estimate reserve demand for the US over the post-GFC period. They assume a log-linear functional form for $v'_R - \varphi$ as a function of reserves and deposits (where reserves as noted are excess reserves over reserve requirements):

 $v'_{R}(Reserves, Deposits) - \varphi = a + b * \ln(Excess Reserves) + c * \ln(Deposits) + u.$

²¹ From the fact that the Federal Reserve has stated its intent to not hold MBS in the longer-run, I infer that such holdings are also politically sensitive and do not explore backing reserves with MBS. Government-backed MBS are generally thought to have some convenience yield but one that is lower than Treasuries. He and Song (2022) estimate an MBS convenience yield about half as large as that on Treasuries.

u is an unobserved reserve demand shock. Using the effective federal funds rate for the short market rate r, the reserve demand from equation (1) then takes the form²²

$$EFFR - IOR = a + b * \ln(Excess Reserves) + c * \ln(Deposits) + u.$$
(6)

LS-VJ instrument excess reserves with the sum of reserves and take-up at the overnight reverse repo (ONRRP) facility. This is needed because a shock to reserve demand affects the split between reserves and ONRRP but is unlikely to affect the sum which thus serves as a useful instrument. The reduced form of the IV estimation regresses the EFFR-IOR spread directly on the exogenous variables

$$EFFR - IOR = A_R + B_R * ln(Reserves + ONRRP) + C_R * ln(Deposits) + U.$$
(7)

LS-VJ emphasize the need to control for deposits to achieve a stable reserve demand function because of strong growth in deposits over the post-GFC sample.²³ In Chart 2, Panel a, left graph illustrates the expected negative correlation between EFFR-IOR and excess reserve supply, consistent with a downward sloping demand curve. However, the relation is unstable as the EFFR-IOR spread is much larger in September 2019 than it was the last time excess reserves had a similar value. Accounting for growing deposits reconciles this instability and results in a stable reserve demand function. Updating LS-VJ's estimates to cover the period 2009M1-2023M4 results in $\widehat{A_R} = -2.187$, $\widehat{B_R} = -0.172$, $\widehat{C_R} = 0.367$, with all parameterestimates significant at the 1% level (accounting for autocorrelation up to monthly 12 lags). The fit is good with an R^2 of 0.89. Reorganizing equation (7), they express the EFFR-IOR spread as a function of the quantity of Reserves+ONRRP, adjusted for the demand shifter (deposits) as follows

$$EFFR - IOR = A_R + B_R * ln \underbrace{\left[(Reserves + ONRRP) * Deposits^{\frac{C_R}{B_R}} \right]}_{Deposit-adjusted Reserves + ONRRP supply} + U.$$
(8)

Chart 2, Panel a, right graph, shows this relation. I graph the EFFR-IOR spread and its fitted value against the deposit-adjusted Reserves+ONRRP supply, $(Reserves + ONRRP) * Deposits^{\frac{C_R}{B_R}}$. The fitted line is the empirical version of $v'_R(.) - \varphi$, expressed as a function of its exogenous drivers.

²² Expressing reserve demand as a function of reserve demand drivers, this corresponds to a reserve demand function given by *Excess Reserves* = α *Deposits*^{β} $e^{\gamma * [EFFR-IOR]} \varepsilon$, where $\alpha = e^{-a/b}$, $\beta = -c/b$, $\gamma = 1/b$, and $\varepsilon = e^{-u/b}$. LS-VJ use total bank deposits (of all commercial banks in the US), but results are similar when using only liquid deposits.

²³ See Lopez-Salido and Vissing-Jorgensen (2023) for results that instrument for deposits. Results do not change much so, for simplicity, I do not instrument for deposits here. What matters is controlling for deposits, but not instrumenting for deposits.

Chart 2

US: Estimating reserve and Treasury demand functions

a) Reserve demand

(Monthly data, 2009M1-2023M4)





(Annual data, 1919-2023)

0

ò

.2







.4 .6 .8 Debt/GDP-Fed/GDP **1**0446

1

1.2





c) Treasury holdings of the Federal Reserve and foreigners

(Annual data, 2019-2023)



Note: Panel a): All data are from FRED.

Panel b): For simplicity and ease of replication, in this paper, I use Debt/GDP at book value, as opposed to at market value as in KVJ. The Debt/GDP series used is from Henning Bohn's homepage up to 2008. I update it using data on Treasury debt from the Federal Reserve's Z1 release, Table L.210, line 1 minus line 10. Line 10 refers to Treasury holdings by federal government defined benefit pension plans. I subtract these for comparability with Bohn's data. See Krishnamurthy and Vissing-Jorgensen (2012) for data sources for the Aaa-Treasury spread. Data on Treasury debt and holdings of the Federal Reserves and foreigners are as of the government's fiscal year end (June or September depending on the year), with GDP measured as the 4-quarter value leading up to the fiscal year end. For 2023, quantity data are for 2023Q1. Spread data are as of the following month.

Panel c): Data on Federal Reserve Treasury holdings are from the Z1 release from 1945 onward and from Banking and Monetary Statistic pre-1945. Data on foreign Treasury holdings are from the Z1 release from 1945 and assumed close to zero before 1945.

5.3 Estimating Treasury demand

Krishnamurthy and Vissing-Jorgensen (2012) estimate v'_T (Treasuries/GDP) by relating the Aaa-Treasury spread to Treasuries/GDP. They consider both a log-linear functional form and a piecewise-linear functional form with a horizontal asymptote. The latter allows for estimation of the (typical) credit risk component of the spread as the horizontal asymptote as Treasury supply gets very large. An asymptote can also be imposed when using a log-linear functional form and I follow that approach here so both the reserve demand and Treasury demand estimations are based on loglinear functional forms.²⁴

KVJ's estimate of $v'_T\left(\frac{Treasuries}{GDP}\right)$ is an estimate of the aggregate demand for Treasuries. What is needed for convenience-maximization is the relation between the Aaa-Treasury spread and private holdings of Treasuries to GDP, $v'_T\left(\frac{Treasuries^{Private}}{GDP}\right)$.²⁵ The two are not the same. For example, suppose the Federal Reserve purchases a lot of Treasuries for macroeconomic reasons unrelated to Treasury convenience. This will appear as an aggregate shock to the demand for Treasuries and will shift v'_T (Treasuries/GDP) to the right. By contrast, the function $v'_T\left(\frac{Treasuries^{Private}}{GDP}\right)$ would not be directly affected by the Federal Reserve's

²⁴ One could also impose an asymptote in the estimation of reserve demand. It would capture the default risk in federal funds, minus φ . Imposing an asymptote could potentially result in a better fit if reserve demand becomes fully satiated over the estimation period. I have estimated EFFR-IOR=max($A_R + B_R * ln(Reserves + ONRRP) + B_R * ln(Deposits), B_R) + U$. The fit is even better than for equation (7) with an R^2 of 0.97. Estimates of A_R , B_R and C_R do not differ much and the lower bound D_R (which is estimated to be -16 bps) only binds in 10 months, all in 2013 and 2014. Accounting for potential saturation of reserve demand therefore does not affect convenience-maximization results substantially.

²⁵ From section 4.3, what is needed is the convenience yield function for the non-bank part of the private sector. I use the Treasury holdings of the overall private sector because the split between banks and non-banks may be correlated with the error term. One can think of this approach as the reduced form of an instrumental variable approach where non-bank Treasury holdings are instrumented by overall private sector holdings.

purchases but the input $\frac{Treasuries^{Private}}{GDP}$ would be reduced by larger holdings of the Federal Reserve. I illustrate these patterns in Chart 2, Panel b. The top left graph shows the main figure from KVJ relating the Aaa-Treasury spread to the total supply of Treasury debt relative to GDP. The top right graph adds data for 2009-2023. The new data points lie far to the right of the earlier relation, partly due to large Federal Reserve purchases over the later period. As shown in the bottom left graph, $v'_T \left(\frac{Treasuries^{Private}}{GDP}\right)$ appears somewhat more stable over time, though the post-GFC points still plot somewhat to the right of the pre-GFC relation, suggesting that even $v'_T \left(\frac{Treasuries^{Private}}{GDP}\right)$ may have shifted to the right. The rightward shift of $v'_T \left(\frac{Treasuries^{Private}}{GDP}\right)$ is mainly due to a positive demand shock to Treasury holding by foreigners (remember that "private" refers to anyone but the Federal Reserve). This is clear from the bottom right figure in which I have subtracted both Federal Reserve and foreign Treasury holdings on the horizontal axis.

The magnitude of Federal Reserve plus foreign holdings of Treasuries is shown in Chart 2, Panel c, left. Federal Reserve plus foreign holdings of Treasuries grew from 17% of GDP in 2007 to 59% in 2021 before falling somewhat by 2023. As a result, Treasury holdings to GDP for other sectors grew less dramatically than total Treasury debt to GDP, illustrated in the right graph.

With only aggregate data to work with, it is not possible to statistically estimate precisely how the $v'_T \left(\frac{Treasuries^{Private}}{GDP}\right)$ function (mapping to Chart 2, Panel b, bottom left) may have changed over time. I assume the reason that the post-GFC points plot further to the right is due to a rightward shift in the relation driven by shocks to foreign demand (or that of other non-Fed sectors), with no change in the default component of the spread (the asymptote).

Using annual data for 1919-2023, I estimate the following relation using non-linear least squares:²⁶

 $y^{Aaa} - y^{Treasury}$

$$= \max \left(A_T + B_T * ln\left(\frac{Treasuries^{Private}}{GDP}\right) + \sum_{i=2009}^{2023} \beta_i D(year = i), C_T \right) + U.$$
(9)

The max operator accounts for saturation of Treasury demand, with C_T the estimated default component of the spread. By including year dummies for 2009-2023 (thus perfectly matching data for these years), I capture the rightward shift post-GFC. This results in $\widehat{A_T} = -0.219$, $\widehat{B_T} = -0.933$, $\widehat{C_T} = 0.306$ and $\widehat{\beta}_{2023} = 0.620$, with $R^2 = 0.908$ ($R^2 = 0.889$ even on the original sample from 1919-2008 where no data points are fitted perfectly by construction). Accounting for autocorrelation up to 10 annual lags, $\widehat{B_T}$, $\widehat{C_T}$ and $\widehat{\beta}_{2023}$ are significant at the 1% level while $\widehat{A_T}$ is not significant.

A potential issue to note is that if the Federal Reserve was to lend out many of the Treasuries it held, this would return some of the liquidity services of those

²⁶ The regression uses fiscal year-end data, September in recent years. Data for $\frac{Treasuries^{Private}}{GDP}$ is not yet available for September 2023. For 2023, I use the value of $\frac{Treasuries^{Private}}{GDP}$ as of the end of 2023Q1 which is 0.759 and the average Aaa-Treasury spread in April 2023 which is 0.658.

Treasuries. *Treasuries*^{Private} = *Treasuries* – *Treasuries*^{Fed} may then understate the Treasury convenience available to the private sector. For example, if the Federal Reserve lent Treasuries out against other less special Treasuries (as in the SOMA Securities Lending Program), this could result on some increase in convenience supply. It is unclear whether the Federal Reserve could -- politically -- lend out large amounts of Treasuries against non-cash non-Treasury collateral. Which collateral would be accepted, on what terms? How large benefits would issuers of such securities get? This would raise the same issues that makes it difficult for the Federal Reserve to hold non-Treasury assets directly. Furthermore, regardless of the collateral posted, securities lending would not transfer the safety benefits of the Treasuries borrowed to those borrowing them (since ownership and principal/coupon payments would remain with the Federal Reserve). Therefore, the convenience-maximizing reserve supply would still be lower with reserves supplied via Treasury purchases and associated Treasury lending than via inconvenient assets.²⁷

5.4 Convenience-maximizing reserve supply

Case A. Reserves supplied via inconvenient assets

Chart 3, Panel a shows what the convenience-maximizing Reserve+ONRRP supply for the US would be as of April 2023, if the Federal Reserve supplied reserves with inconvenient assets. I graph the fitted value for the EFFR-IOR spread based on equation (7)

$A_R + B_R * \ln(Reserves + ONRRP) + C_R * \ln(Deposits)$

with $\widehat{A_R} = -2.187$, $\widehat{B_R} = -0.172$, $\widehat{C_R} = 0.367$, and using the value of deposits for April 2023 which is \$17.258T. A value of Reserves+ONRRP of \$3.285T sets the predicted spread, and thus the predicted value of $v'_R(R, .) - \varphi$, to zero. The gray shaded area in the figure indicates the range of data over which the regression was estimated. Chart 3, Panel b graphs the time series evolution of the convenience-maximizing Reserves+ONRRP supply for this case. The value grows over time as deposits grow. The actual Reserves+ONRRP supply exceeded the convenience-maximizing Reserves+ONRRP value throughout the sample, except for a period in 2019 and early 2020.

²⁷ See Roh (2022) for an analysis of securities lending in the context of QE. Roh shows that for both the Federal Reserve and the ECB, on average less than 2% of bonds held during government bond QE have been lent out.

Chart 3



US: Convenience-maximizing reserve supply if reserves are supplied via holdings of inconvenient assets (Case A)

b) 2009M1-2023Q4



Note: In panel a), grey shading indicates range of data used in estimation. The red line is constructed using Reserves+ONRRP values between \$100B and \$7T.

These calculations do not account for the fact that deposits may themselves be a function of Reserves+ONRRP supply. A more ambitious approach would model this dependence. To the extent that a smaller balance sheet size causes lower deposits, the convenience-maximizing Reserves+ONRRP supply will be lower than calculated here. This effect is likely to be modest given the estimates of Diamond, Jiang and Ma (2022) that only 7 cents of a dollar of extra reserves is funded with additional deposits.

Case B. Reserves supplied via Treasury holdings

Chart 4 turns to the more realistic case for the US (in the longer run) in which reserves are supplied via Federal Reserve holdings of Treasuries. In Chart 4, Panel a, I graph in red the estimated convenience yield of reserves as a function of Reserves+ONRRP as of April 2023 (this is the same line as the one in Chart 3, Panel a). In blue, I graph the estimated convenience yield on Treasuries as a function of the private sector's holdings of Treasuries as of April 2023. This is based on the estimated version of equation (9). Specifically, the blue line is

$$\max\left(A_T + B_T * ln\left(\frac{Treasuries^{Private}}{GDP}\right) + \beta_{2023} - C_T, 0\right)$$

with $\widehat{A_T} = -0.219$, $\widehat{B_T} = -0.933$, $\widehat{C_T} = 0.306$, $\widehat{\beta}_{2023} = 0.620$, and using GDP for the four quarters 2022Q2-2023Q1 of \$25.899T. It is apparent that the demand for Treasuries is much larger than the demand for reserves in the sense that Treasuries remain convenient for much larger supply (presumably because a much wider set of entities can hold Treasuries than can hold reserves and because Treasuries are available for many maturities). Private Treasury demand is saturated for a total Treasury supply (minus Federal Reserve holdings) a bit below \$30T.

In Chart 4, Panel a, point A on the red line indicates the predicted EFFR-IOR spread in the reserve market, given current reserves+ONRRP supply as of April 2023 of 5.554T. Point A on the blue line indicates what the predicted Aaa-Treasury spread would be, if the Federal Reserve exclusively held Treasuries, so *Treasuries*^{Private} = *Treasuries* – *Treasuries*^{Federal Reserve}. Current total Treasury supply is \$24.614T (end of 2023Q1 from the Z1 release, L.210, line 1 minus line 10). Federal Reserve holdings are \$4.961T, but they would be \$8.601T if the Federal Reserve held only Treasuries (as current MBS holdings are large). This would result in private Treasury holdings of \$16.013T at point A on the blue line. The black vertical line indicates current *Treasuries*^{Private} given that the Federal Reserve currently holds a mix of Treasuries and MBS, unlike in its longer-run plans.

Since point A on the red line is a lot lower than point A on the blue line, conveniencemaximization implies lowering the supply of Reserves+ONRRP thereby increasing *Treasuries*^{Private} correspondingly. Point B on the red and blue lines illustrate the resulting outcome. Convenience yields are now equalized at 29 bps. This happens for reserves+ONRRP=\$598B, resulting in private Treasury holdings of \$20.969T. The convenience-maximizing supply of Reserves+ONRRP is calculated as the value *x* that solves

$$\begin{aligned} \widehat{A_R} &+ \widehat{B_R} * \ln(x) + \widehat{C_R} * \ln(Deposits) \\ &= \max\left(\widehat{A_T} + \widehat{B_T} * \ln\left(\frac{Treasuries - [x + Autonomous factors]}{GDP}\right) + \widehat{\beta}_{2023} - \widehat{C_T}, 0\right) \end{aligned}$$

given the values of deposits, Treasuries, autonomous factors, and GDP (with April 2023 data for deposits and autonomous factors, end of 2023Q1 data for Treasuries, and 2022Q2-2023Q1 data for GDP). Chart 4, Panel b, illustrates how the convenience yield on reserves (net of balance sheet costs) and on Treasuries change with the supply of reserves+ONRRP.

Chart 4





a) Reserves and Treasury convenience yields using latest available data





c) Convenience-maximizing reserves+ONRRP supply, 2009-2023



Chart 4, Panel c, illustrates the time-series evolution of the convenience-maximizing supply of Reserves+ONRRP. The value is tiny in early years because of high Treasury scarcity when Debt/GDP was lower. The series fluctuates due to fluctuations in the four inputs (deposits, Treasuries, autonomous factors, and GDP) and due to shifts in the Treasury convenience yield curve which, as explained above, is assumed to shift left/right to perfectly fit the points from 2009-2023.

6

Estimating the convenience-maximizing reserve supply for the euro area

This section repeats the convenience-maximization exercise for the euro area. A key difference will be that, because it appears politically feasible for the ECB to supply reserves via inconvenient asset holdings (bank loans, typically collateralized with inconvenient assets), I will mainly focus estimation of the ECB's convenience-maximizing reserve supply on this case.

6.1 Yield spreads

Chart 5 provides information on yield spreads for the euro area. The top left figure shows the spread between measures of banks' funding costs and the ECB's deposit facility rate. These spreads are similar to the EFFR-IOR spread for the US. The EONIA-DFR spread is just over 100 bps up to the financial crisis, a reminder that the ECB differed from the Federal Reserve over this period by not setting the interest rate on reserves to zero and instead operating with a reserve scarcity around 100 bps. The ECB moved to less scarce reserves around the start of the financial crisis. There is a spike in EONIA-DFR around the European sovereign debt crisis, likely related to bank default-risk. Rather than trying to estimate this risk, I will estimate reserve demand for the euro area for the 2013M1-2023M4 period.

The ECB started publishing data on the euro short-term rate (ESTR) in October 2019. ESTR includes transactions between banks and non-banks. EONIA was discontinued in January 2022. From October 2019 onward, EONIA was set at a fixed spread of 8.5 bps over ESTR (by the administrator of EONIA). I assume that had ESTR been quoted in earlier years it would have been 8.5 bps below EONIA. By this measure, ESTR (directly measured or assumed 8.5 bps below EONIA) – DFR has been negative about 80% of the time period since 2013M1. This series is shown in red in Chart 5, bottom left.

Chart 5

Euro area: Yield spreads on reserves and government bonds (relative to inconvenient assets)

(Monthly data)



Spreads on government bonds: Germany



Combined chart

Spreads on government bonds: Germany, Netherlands, France



Note: Data for EONIA, ESTR and DFR are from FRED. Data for KfW yields and government bond yields are from Bloomberg.

Chart 5, top right, provides measures of the yield spread on German government bonds, measured relative to euro-denominated KfW bonds. The KfW bonds are real estate-related bonds that have government guarantees. They may thus appeal to safety-investors. Therefore, the KfW-Bund spreads likely understate the convenience yield on bunds, capturing mainly the liquidity part. That said, the KfW-Bund spreads are substantial, more so for longer bonds. At the 10-year maturity, the KfW-Bund spread is 60 bps as of April 2023, while it is 30 bps at the 2-year maturity. Chart 5, bottom right shows that positive convenience yields in the euro area is not entirely unique to German government bonds. As an example, 10-year Dutch government bonds also carry a convenience yield, while the convenience yield on 10-year French government bonds is slightly positive in some recent years.

The combination of a negative ESTR-DFR spread and positive KfW-Government bond spreads for Germany and (to a lesser extent) a few other euro area countries suggests that, from a convenience-maximization perspective, the ECB's asset choice matters. The convenience-maximizing ECB reserve supply is larger if the ECB supplies reserves exclusively via inconvenient assets.

6.2 Estimating reserve demand

Using data for 2013M1-2023M4, I repeat the estimation from Lopez-Salido and Vissing-Jorgensen (2023) using data from the euro area.

$$ESTR - DFR = a + b * \ln(Excess Liquidity) + c * \ln(Deposits) + u$$
(10)
= a + b * ln [(Excess Liquidity) * Deposits $\frac{c}{b}$] + u

where I measure excess liquidity (ECB terminology for excess reserves) as banks' total holdings with the ECB (current account holdings+deposit facility holdings) minus required reserves. The variable $\left[(Excess Liquidity) * Deposits^{\frac{c}{b}}\right]$ is the ECB's deposit-adjusted excess liquidity supply. Since the ECB does not operate an ONRRP facility, the issue of endogeneity of reserves in response to reserve demand shocks in the presence of such a facility does not emerge. Therefore, I do not instrument for excess liquidity (nor deposits).

For the euro area, the fit is slightly better when controlling for overnight deposits rather than total deposits, so I use overnight deposits. I estimate $\hat{a} = -0.420$, $\hat{b} = -0.064$, and $\hat{c} = 0.095$. *b* and *c* are significant at the 1% level, accounting for autocorrelation up to 12 monthly lags, while *a* is not significant. The R^2 is 0.87.

Chart 6, right, graphs the ESTR-DFR spread and the fitted value against the depositadjusted excess liquidity supply. The fit is good, though one could experiment with other functional forms to get more curvature for low deposit-adjusted excess liquidity supply.

Chart 6

Euro area: Estimating reserve demand function



Note: Excess liquidity=Current account balances+Deposit facility-Required reserves. Data on deposits and the inputs for calculating excess liquidity are from the ECB's webpage.

6.3 Convenience-maximizing reserve supply

Case A. Reserves supplied via inconvenient assets

Chart 7, Panel a estimates the convenience-maximizing excess liquidity supply as of April 2023, given overnight deposits of €9.379T and assuming that liquidity is supplied via inconvenient asset holdings. I estimate a value of excess liquidity of €1.251T. Adding required reserves of €165B gives a convenience-maximizing liquidity estimate of €1.416T.

Chart 7



Euro area: Convenience-maximizing reserve supply if reserves are supplied via holdings of inconvenient assets (Case A)





Note: In panel a), grey shading indicates range of data used in estimation. Red line is constructed using excess liquidity values between EUR 50B and EUR 6T. In panel b), Excess liquidity=Current account balances+Deposit facility-Required reserves. Deposit data are from the ECB's webpage.

This estimate is likely somewhat higher than the true convenience-maximizing value since the blue fitted line in Chart 6, right, is a bit to the right of the data around a spread of zero. Again, working on the functional form more would improve the accuracy of the estimate.

Chart 7, Panel b, left shows in blue how the estimated convenience-maximizing excess liquidity has evolved since the start of 2013. The estimate increases from

under €400B in 2013M1 to around €1.25T in 2023M4. The time-series variation in the estimate is driven by the time-series variation in reserve demand due to changing overnight deposits, shown in bright green in Chart 7, Panel b, right. Overnight deposits increase from €4.3T in 2013M1 to €9.4T in 2023M4. By contrast, omitting deposits from the regression in (10) would incorrectly lead to an estimated convenience-maximizing excess liquidity of around €600B for all years.

In the case of the euro area, omitting deposits from the regression in (10) has only a modest effect on the R^2 . This is likely due to ln(Excess liquidity) and ln(Deposits) being highly correlated over the 2013M1-2023M4 sample, with a correlation of 0.94.²⁸ However, as discussed, omitting ln(Deposits) leads one to miss the fact that the convenience-maximizing excess liquidity grows over time with the size of the banking sector. While I have focused my estimations on the Federal Reserve and the euro area, Bank of England (2023) implements the LS-VJ reserve demand estimation for the UK and also finds an important role for deposits. For the UK, deposit growth from 2018 to 2022 leads to sharply rising estimates over time in the value of reserves at which reserve scarcity turns positive.

Case B. Reserves supplied via convenient assets

Since the ECB can supply reserves via inconvenient assets, it is less relevant to calculate the convenience-maximizing reserve supply for the euro area if reserves were to be supplied mainly via holdings of government bonds. Furthermore, attempting to estimate this number is complicated by the fact that we do not have a long time series of data with which to estimate the convenience yield function for euro-denominated convenient government bonds, notably Bunds (their convenience yield may have changed with the introduction of the euro). Government bond supplies move slowly and there is little QE-induced variation in the supply of bunds to the private sector given that the ECB started QE only in 2015.

That said, since I was asked for this number, here is a back-of-the-envelope attempt at calculating the convenience-maximizing reserve supply for the euro area with reserves supplied via government bond purchases.

Under the simplifying assumption that only German bonds have convenience yields, from section 4.2 we have that the convenience-maximizing reserve supply solves

$$v_R'(R) - \varphi = v_B'(B_1^{priv}) * \omega * \alpha_1.$$

Suppose the ECB supplied reserves solely via holdings of euro area government bonds, so the weight for government bonds ω equals 1 for this hypothetical scenario. Suppose also that the ECB purchased government bonds in proportion to the ECB capital key. This key is 21.4% for Germany, so set $\alpha_1 = 0.214$.

From Chart 5, top right, the convenience yields on bunds vary by horizon, but as an average suppose it is around 40 bps. Then $v'_B(B_1^{priv}) * \omega * \alpha_1$ is around 8 bps.

²⁸ This compares to a correlation of 0.86 of ln(Reserves+ONRRP) and ln(Deposits) for the US over the 2009M1-2023M4 period.

Therefore, the value of $v'_R(R) - \varphi$, proxied by the ESTR-DFR spread, that corresponds to convenience-maximization in this scenario is at most 8 bps. Why at most? Consider Chart 4, Panel a as applied to this case. The blue line would be $v'_B(B_1^{priv}) * \omega * \alpha_1$. If point A on the blue line was at 8 bps, then the points B on the red and blue lines are below 8 bps. Point B on the red line, corresponding to a spread of 8 bps based on the reserve demand estimation for the euro area, can be calculate from the red line in Chart 7, Panel a. It solves

 $0.08 = a + b * \ln(Excess Liquidity) + c * \ln(Deposits)$

using current overnight deposits of €9.4T. The resulting value for excess liquidity is €356B. Adding required reserves of €165B, we get €521B (of which €111B in bunds). That is an underestimate since the convenience yield on bunds would decline as the ECB's bund holdings were reduced from the current level of about €750B. Therefore, the convenience-maximizing liquidity supply for the euro area for case B equalizes $v'_R(R) - \varphi$ and $v'_B(B_1^{priv}) * \omega * \alpha_1$ at a number below 8 bps, corresponding to a liquidity supply above €521B but below the value of €1.4T from case A where the ECB supplies reserves via inconvenient assets.

7 Conclusion

Reserves constitute a liquid and safe asset to the public. The paper provides a simple framework for thinking about central banks' provision of reserves in terms of convenience yields. The framework is intended to guide central bank balance sheet choice when short rates are above the ELB so the balance sheet is not needed for stimulus or contractionary purposes.

If the central bank supplies reserves via holdings of assets without convenience yields (case A), the convenience-maximizing reserve supply sets the reserve convenience yield net of balance sheet costs to zero. By contrast, if the central bank supplies reserves via holdings of government bonds that themselves carry a convenience yield due to liquidity or safety (case B), then the convenience-maximizing reserve supply is smaller and equalizes the net convenience yield on reserves and the convenience yield on government bonds.

Political constraints on which assets a central bank can hold without it creating threats to its independence therefore affect its convenience-maximizing reserve supply. I argue that the Federal Reserve fits case B better while the ECB could likely choose to be in case A. I provide estimates of the convenience-maximizing reserve (or reserve+ONRRP) supply for the US and the euro area.

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